Explanation Systems for Approval-Based Multiwinner Voting

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Setting & Motivation

Approval-Based Multiwinner Voting

- Set of n voters V and m candidates C. Each voter $i \in V$ approves a subset of candidates $A_i \subseteq C$.
- We want to **explain** how any given committee $W \subseteq C$ represents the approval profile, **independent of how it was selected**:
  - How over- or underrepresented is each voter?
  - How did individual voters influence the selected candidates?
  - Why was a certain candidate (not) chosen?

Proportional Representation

A central theme in the literature is **proportionality**: A group of $x\%$ of the voters should decide on $x\%$ of the committee.

But: Previous axioms mostly yield binary yes-or-no answers, i.e., they state whether the committee is proportional (under some given notion).

Towards More Fine-Grained Explanations

Goal: Quantify proportionality at the voter level and how W represents the given approval profile.

Building on [PeSk20], we explain committees via **price systems:**

- **Budget** b_i: total influence of voter i on the outcome.
- **Payment** $p(i, c)$: voter i's contribution to the selection of candidate $c \in W$.
- **Residual** r_i: voter i's unexercised influence.
- Each candidate $c \in W$ is bought at a price of 1, and no unselected candidate can be afforded by their supporters' residuals (residual stability).

Previous work uses price systems as a **certificate for binary proportionality notions.** We extend this framework:

- Handle **any given committee** by allowing approximation through **non-uniform budgets**.
- Focus on deriving **meaningful, interpretable price systems**.

Axiomatizing Good Explanations

We propose **axiomatic desiderata** for **interpretable price systems**:

- **Structural coherence:** Price systems should be interpretable, so **differences in payment reflect structural differences**.
- **Faithful representation of influence:** We **restrict residuals and payments** to ensure that they reflect the voters' power.
- **Alignment with proportionality:** Proportional committees yield uniform budgets; **violations are reflected through non-uniform budgets**.

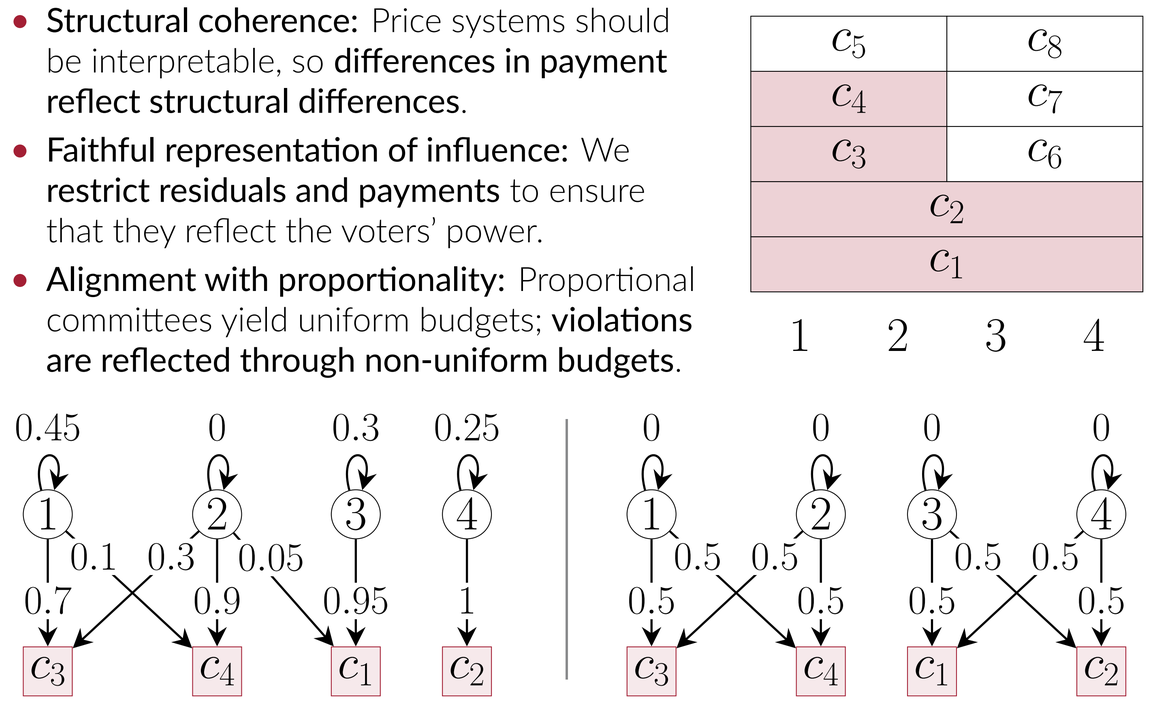


Figure 1. Approval profile and committee W, and two price systems for W.

Symmetry. The price system respects all symmetries of the instance, i.e., it is invariant under all automorphisms.

1-Stability. The price system is residual-stable and there are no candidates $c \in C \setminus W$ and $c' \in W$ such that

$$\sum_{i \in V[c] \setminus V[c']} r_i + \sum_{i \in V[c'] \cap V[c]} p(i, c') > 1.$$

Laminar-Proportional-Uniformity. If the committee W is laminar proportional [PeSk20], then all voters have the same budget.

Computing Explanation Systems

We develop **explanation rules** to compute a price system for **any** committee.

Two baseline rules:

- **Approx. Priceability:** Minimize pairwise budget difference $\sum_{i,j \in V} |b_i - b_j|$ subject to residual-stability.
- **Equal Split:** Split candidate costs evenly among supporters and then distribute additional residual.

Our main rule:

- **Continuous Phragmén:** Voters continuously accumulate and spend budget over time on approved selected candidates, as long as doing so does not destabilize the price system (details are tricky).

Continuous Phragmén satisfies all ten proposed jointly satisfiable axioms and is computable in polynomial time.

Experiments

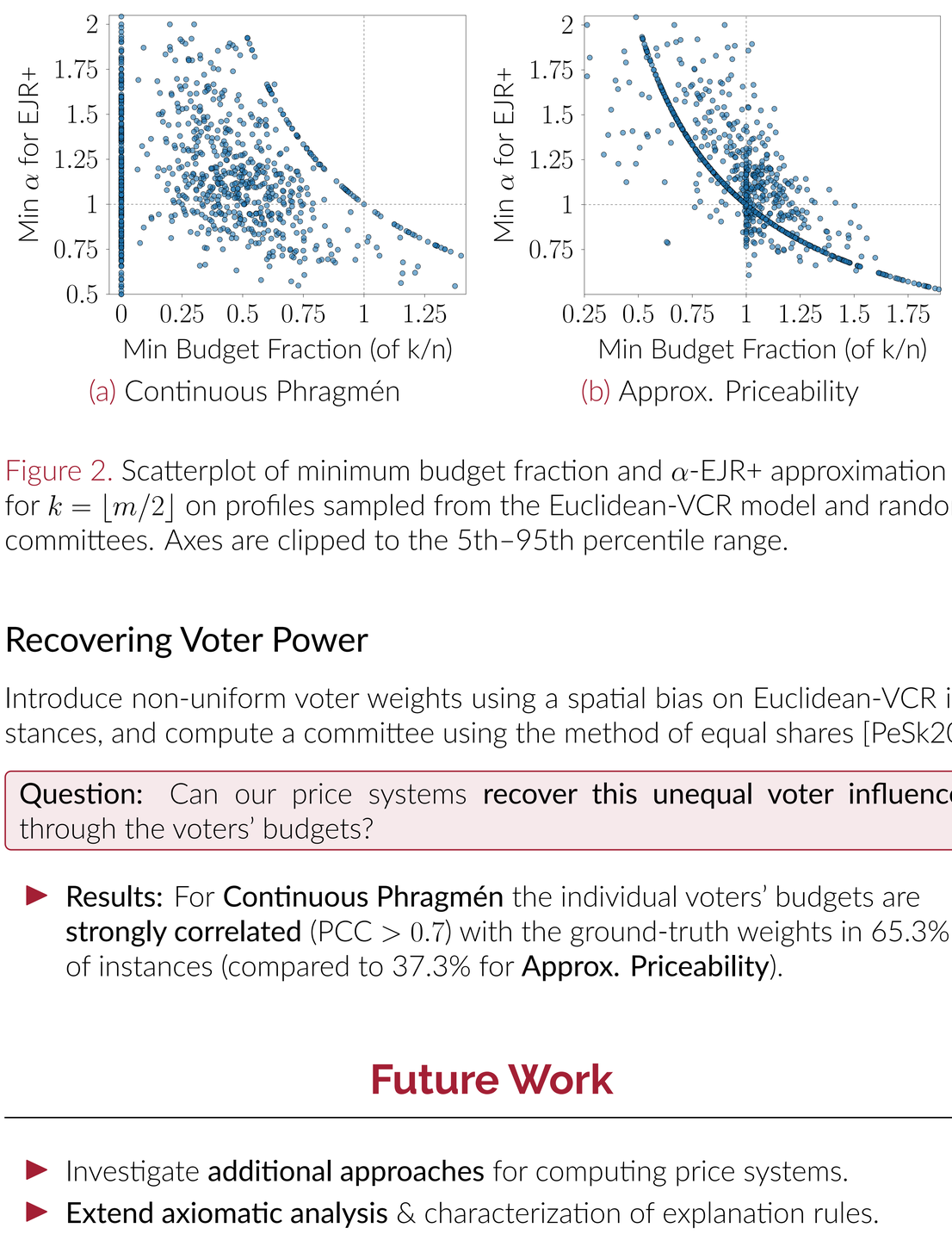
EJR+ and Budgets: Reflecting Disproportionality

Question: Does **non-proportionality under EJR+** result in **non-uniform voter budgets?**

α-EJR+ [BrPe23]. For $\alpha > 0$, a committee W satisfies α-EJR+ if there is no candidate $c \notin W$, group of voters $V' \subseteq V$, and $\ell \in \mathbb{N}$ with $|V'| \geq \alpha \cdot \frac{\ell n}{|W|}$ such that

$$c \in \bigcap_{i \in V'} A_i \text{ and } |A_i \cap W| < \ell \text{ for all } i \in V'.$$

- **Results:** Strong association between the provided α-EJR+ approximation and the minimum budget for **Continuous Phragmén**. Weaker correlation for **Approx. Priceability**.



(a) Continuous Phragmén (b) Approx. Priceability

Figure 2. Scatterplot of minimum budget fraction and α-EJR+ approximation for $k = \lfloor m/2 \rfloor$ on profiles sampled from the Euclidean-VCR model and random committees. Axes are clipped to the 5th–95th percentile range.

Recovering Voter Power

Introduce non-uniform voter weights using a spatial bias on Euclidean-VCR instances, and compute a committee using the method of equal shares [PeSk20].

Question: Can our price systems **recover this unequal voter influence** through the voters' budgets?

- **Results:** For **Continuous Phragmén** the individual voters' budgets are **strongly correlated** (PCC > 0.7) with the ground-truth weights in 65.3% of instances (compared to 37.3% for **Approx. Priceability**).

Future Work

- Investigate **additional approaches** for computing price systems.
- **Extend axiomatic analysis** & characterization of explanation rules.
- Extension to **heterogeneous costs** in participatory budgeting, or to **ranked or cardinal preferences**.
- Apply framework to **audit fairness in real-world subset selection problems**.

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Workshop *Fair & Explainable Collective Decision*
Institut Pascal, Paris

[BrPe23] Brill & Peters, "Robust and Verifiable Proportionality Axioms for Multiwinner Voting", EC '23.

[PeSk20] Peters & Skowron, "Proportionality and the Limits of Welfarism", EC '20.