Tobias Friedrich, Timo Kötzing, Andrew Sutton **Summer Term 2015**

Homework 3 – "Heuristic Optimization"

https://hpi.de/friedrich/teaching/ss15/heuristic-optimization.html

This homework is due **Wednesday, May 13, 12:45**. This homework is to be submitted **at the mailbox titled "Heuristic Optimization" at the reception desk of the main building**.

We use the following definitions on O-notation: Let $g : \mathbb{N} \to \mathbb{R}^+$ be a mapping from the natural numbers to the positive real numbers. We let

$$O(g) = \{f : \mathbb{N} \to \mathbb{R}^+ \mid \exists c > 0 \exists n_0 \in \mathbb{N} \forall n \geq n_0 \colon f(n) \leq c \cdot g(n)\}.$$

Furthermore, we let

$$\Omega(g) = \{f : \mathbb{N} \to \mathbb{R}^+ \mid \exists c > 0 \exists n_0 \in \mathbb{N} \forall n \geq n_0 \colon g(n) \leq c \cdot f(n)\}.$$

Finally, we let $\Theta(g) = O(g) \cap \Omega(g)$. For any set of functions A and any function f we write $f = A$ instead of $f \in A$.

For any set of functions A we can define operations on this set as follows.

$$\begin{aligned} \forall r : A + r &= \{f + r \mid f \in A\}; \\ \forall r : r \cdot A &= \{rf \mid f \in A\}; \\ 2^A &= \{2^f \mid f \in A\}. \end{aligned}$$

Note that when A is a set of functions with positive range, then $-A$ is not, but 2^{-A} is. If there is some n_0 such that something holds for all n larger than n_0, we also say that this something holds "for n large enough".

Exercise 4 *Show the following claims.*

(a) *Let $f : \mathbb{N} \to \mathbb{R}^+$. Suppose that there is a $c \in (0, 1)$ (the open interval from 0 to 1) such that, for all n large enough, $f(n + 1) \leq cf(n)$. Then $f = 2^{-\Omega(n)}$.*

(b) *There is an f with $f = 2^{-\Omega(n)}$ such that there is no $c \in (0, 1)$ such that, for all n large enough, $f(n + 1) \leq cf(n)$.*

(c) *Let $f, g : \mathbb{N} \to \mathbb{R}^+$. If, for all n large enough, $f(n) \leq g(n)$, then $2^f = O(2^g)$.*

(d) *Let $f, g : \mathbb{N} \to \mathbb{R}^+$. If $2^f = O(2^g)$, then $f = O(g)$.*

(e) *There are $f, g : \mathbb{N} \to \mathbb{R}^+$ such that $f = O(g)$ but **not** $2^f = O(2^g)$.*