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Homework 4 – "Heuristic Optimization"

https://hpi.de/friedrich/teaching/ss15/heuristic-optimization.html

This homework is due **Wednesday, May 20, 12:45**. This homework is to be submitted at the mailbox titled "Heuristic Optimization" at the reception desk of the main building.

In Lecture 5, we claimed that there are functions $f\colon \{0,1\}^n \to \mathbb{R}$ for which the runtime of the (1+1) EA is worse than the worst-case runtime of RandomSearch. Define the function $g\colon \{0,1\}^n \to \mathbb{R}$ as follows

$$g(x) = \begin{cases} 1 & \text{if } |x|_1 = n, \\ 0 & \text{if } 3n/4 \le |x|_1 < n, \\ 1/2 & \text{otherwise;} \end{cases}$$

Theorem 1 (Law of Total Probability) *Let $\mathcal{E}$ and $\mathcal{F}$ be mutually disjoint events and $\mathcal{E} \cup \mathcal{F} = \Omega$. Denote as $E(T \mid \mathcal{E})$ the expectation of T conditioned[1] on event $\mathcal{E}$. Then*

$$E(T) = E(T \mid \mathcal{E}) \Pr(\mathcal{E}) + E(T \mid \mathcal{F}) \Pr(\mathcal{F}).$$

Exercise 5 *Answer the following.*

(a) *What is the expected runtime of* RandomSearch *to maximize g?*

(b) *Let $\mathcal{E}$ be the event that the initial solution of the (1+1) EA has at most $n/4$ zero bits. Derive an upper bound on $\Pr(\mathcal{E})$.*
Hint: use one of the methods we discussed in Lecture 5.

(c) *Suppose $\mathcal{E}$ has* ***not*** *occurred. Give a lower bound on the expected time for the (1+1) EA to first generate the all-ones string under this condition.*

(d) *Conclude that the expected runtime to maximize g for the (1+1) EA is worse than the runtime of* RandomSearch *on g.*
Hint: use the law in Theorem 1.

[1] http://en.wikipedia.org/wiki/Conditional_expectation