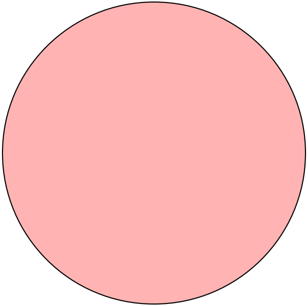
Probabilistic Timed Graph Transformation Systems

Maria Maximova, Holger Giese, Christian Krause

19th July 2017

System's Aspects and Modeling Formalisms

Structure Dynamics



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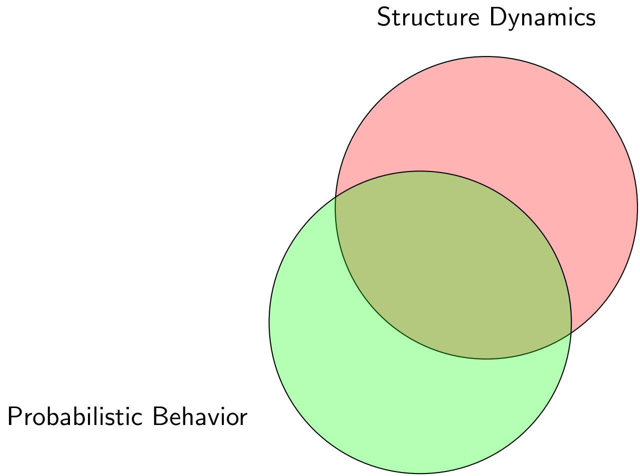

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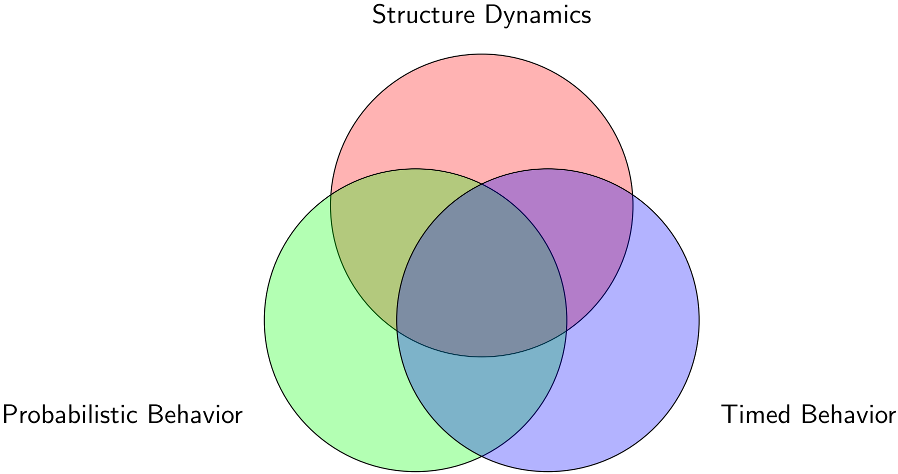

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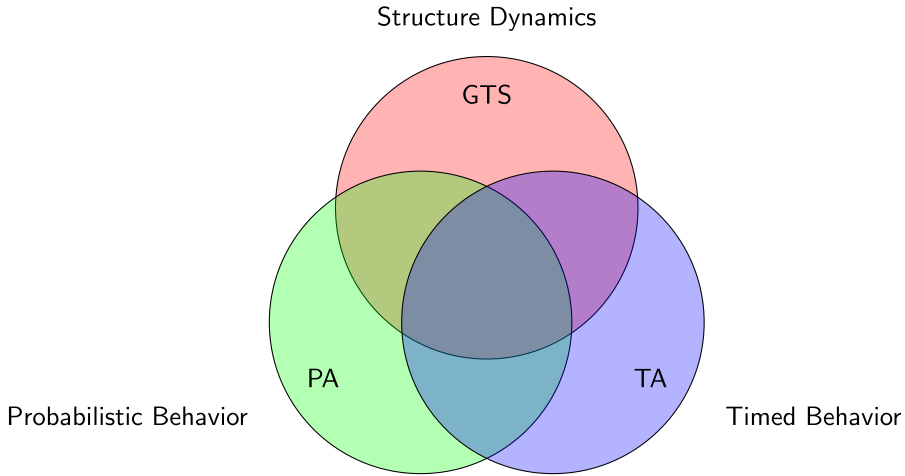

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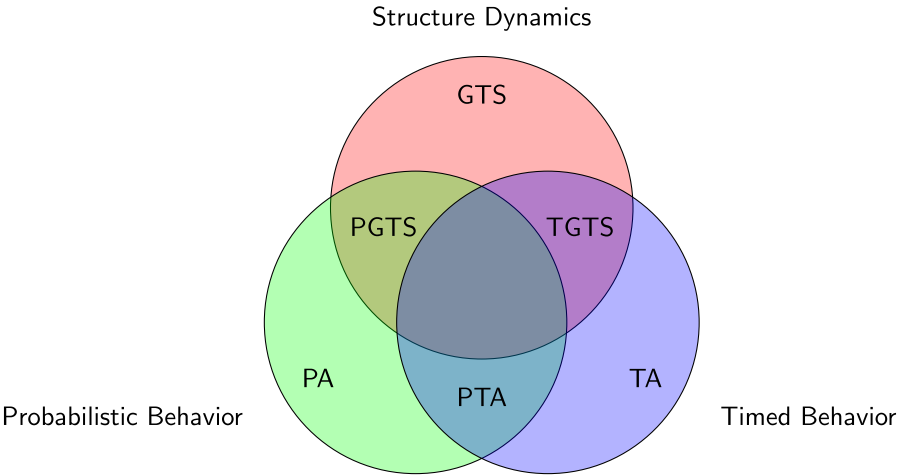

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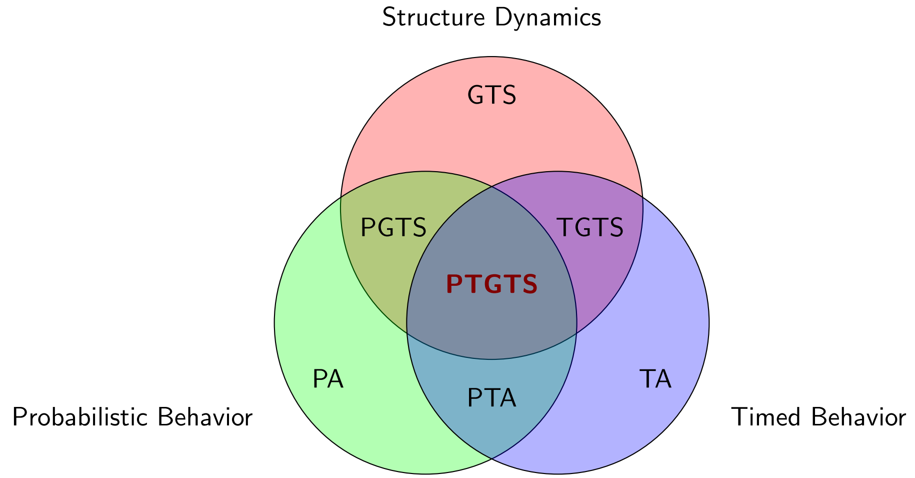

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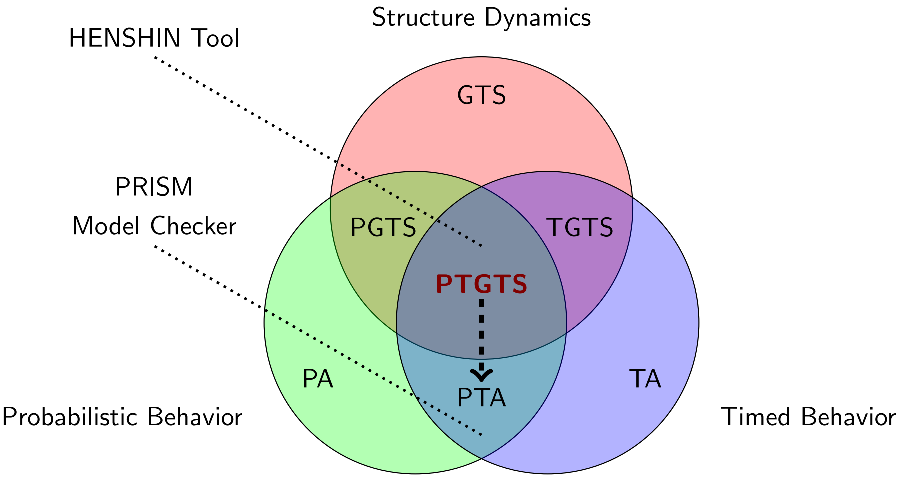

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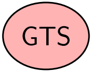

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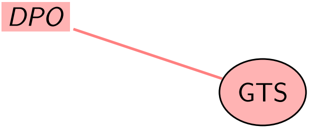

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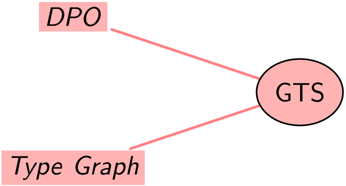

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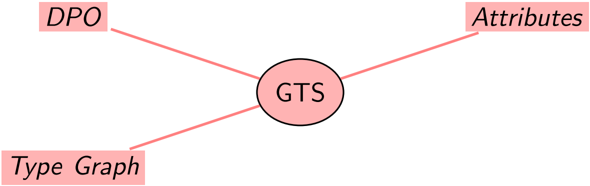

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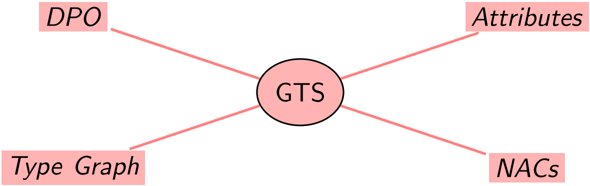

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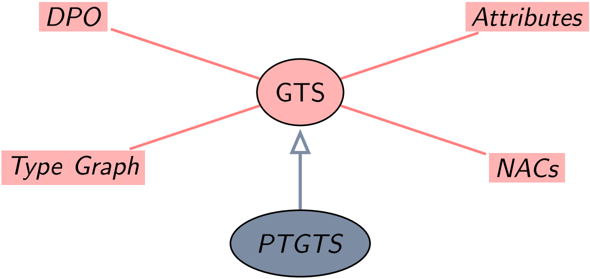

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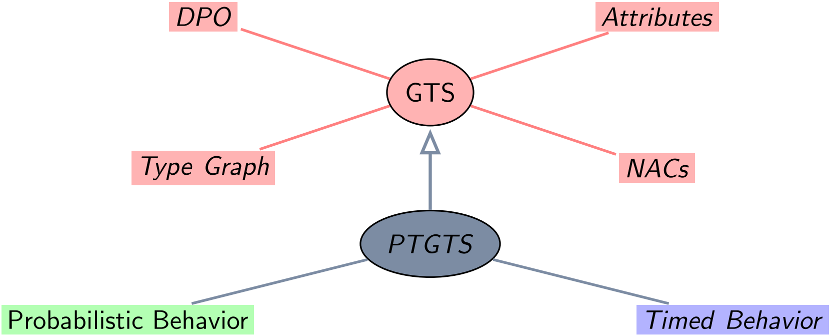

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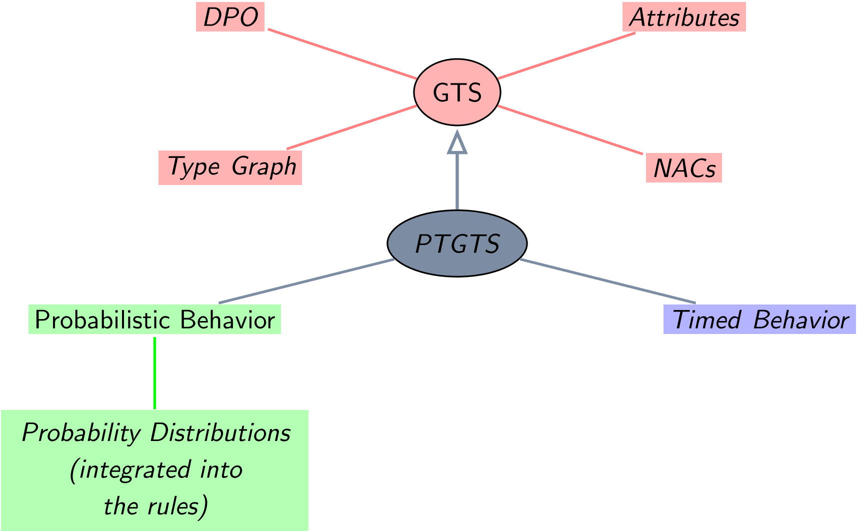

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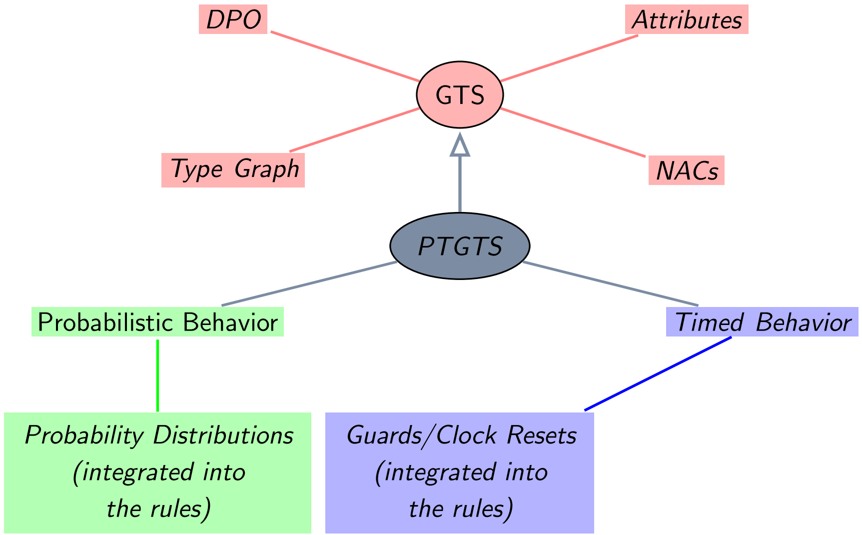

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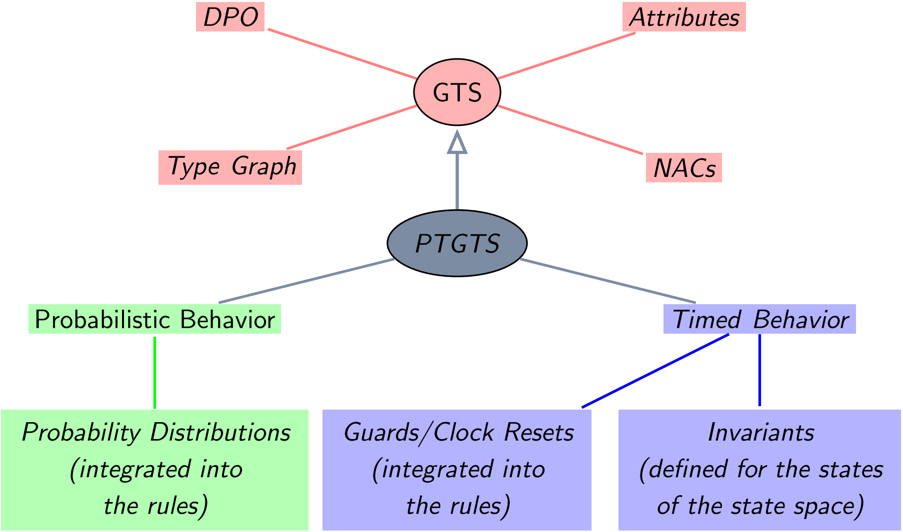

Formal Model

Definition (Probabilistic Timed Graph Transformation System)

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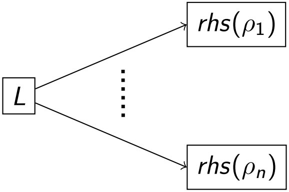

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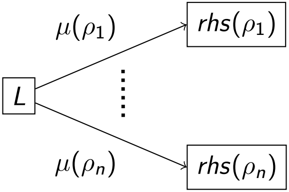

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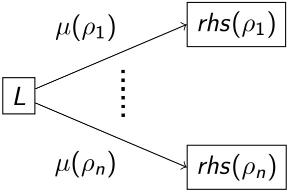

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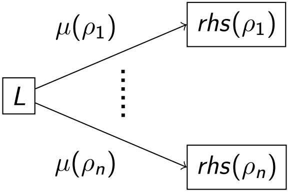
- $r_C \subseteq ClockNodes(L)$ is a set of nodes of the type *Clock* to be reset.



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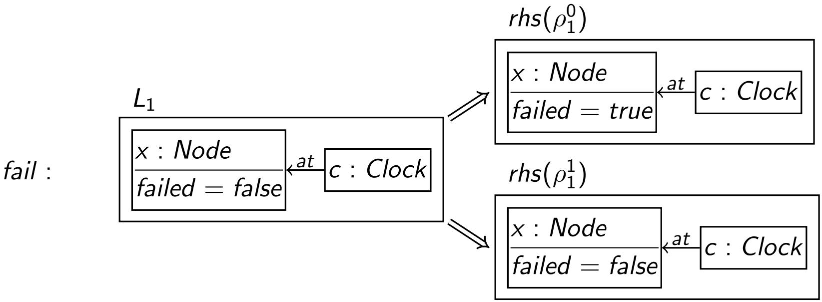
Example for a PTGT Rule



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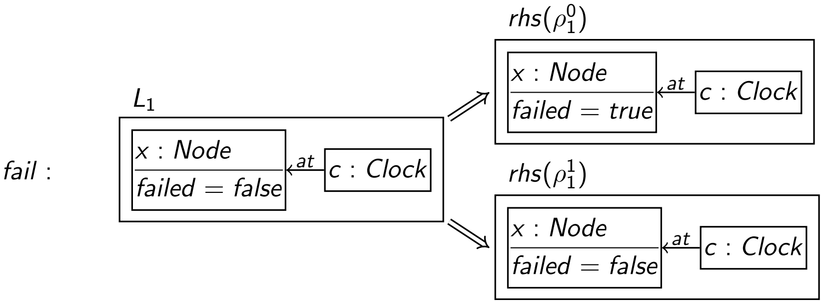

$$
\begin{aligned}
\mathit{fail} &= (L_1, P_1, \mu_1, \phi_1, r_{C_1}) \text{ with} \\
P_1 &= \{\rho_1^0, \rho_1^1\} \\
\mu_1 &= \{(\rho_1^0, 0.1), (\rho_1^1, 0.9)\} \\
\phi_1 &= (c \geq 2) \\
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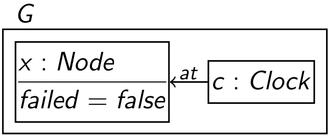
Θ_{fail} :



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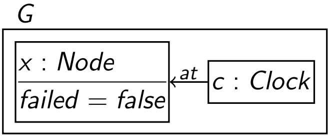

$$\Theta_{fail} = (G, \phi) \text{ with } \phi = (c \leq 5)$$

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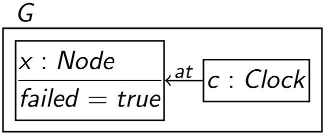

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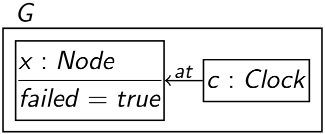

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Semantics of PTGTS

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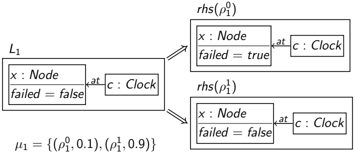

$$\mu_1 = \{(\rho_1^0, 0.1), (\rho_1^1, 0.9)\}$$

$$\phi_1 = (c \geq 2)$$

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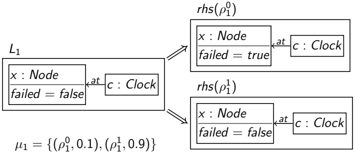

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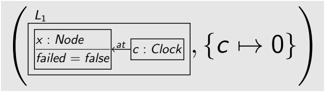

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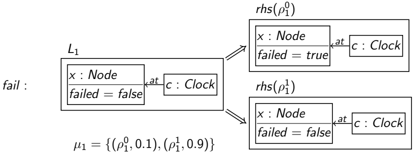

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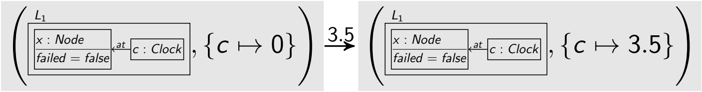

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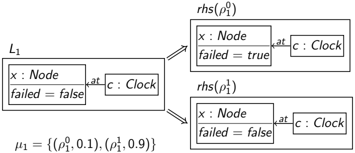

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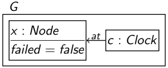

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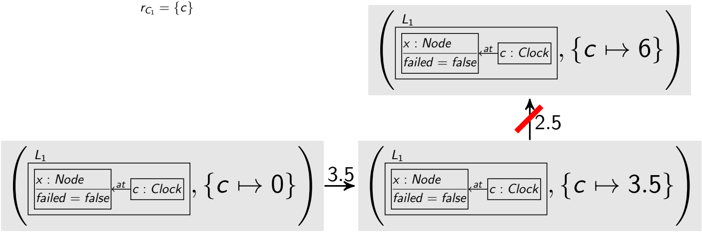

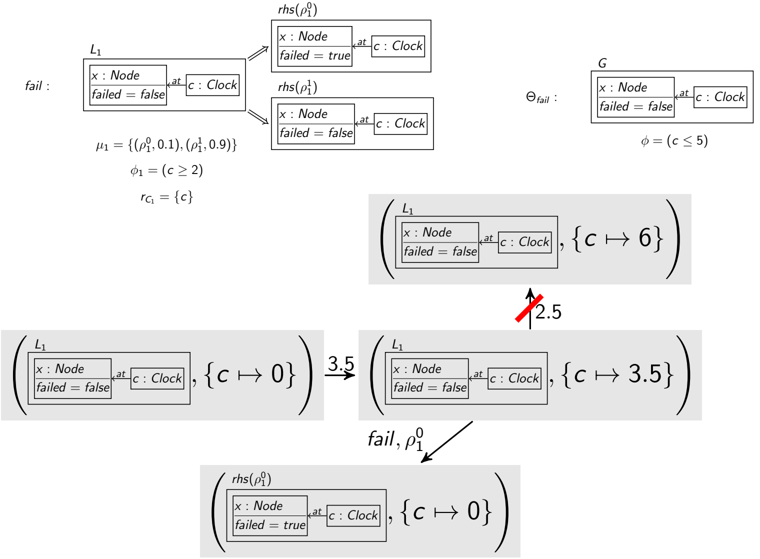

fail :
L_1
x : Node
failed = false
at
c : Clock
$rhs(\rho_1^0)$
x : Node
failed = true
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Θ_{fail} :
G
$\phi = (c \leq 5)$
$\{c \mapsto 0\}$
3.5
$\{c \mapsto 3.5\}$
2.5
$\{c \mapsto 6\}$
$fail, \rho_1^0$

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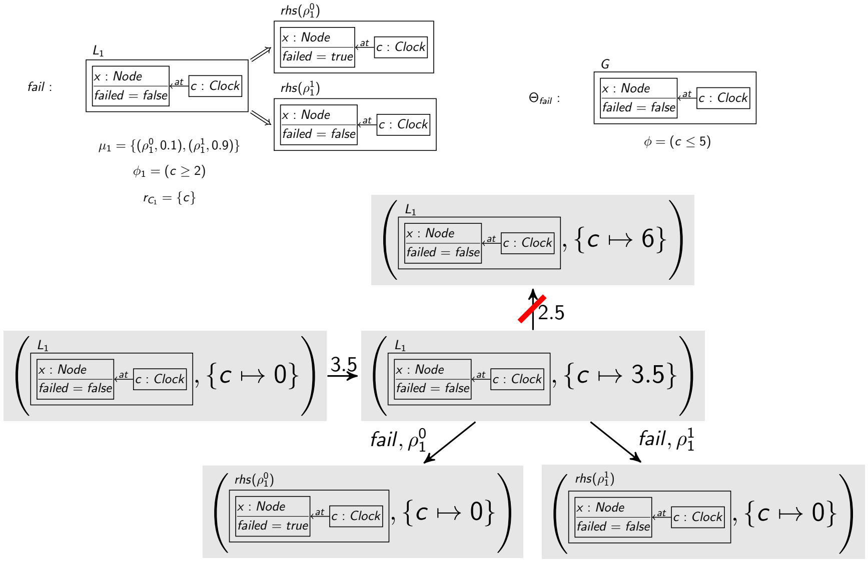

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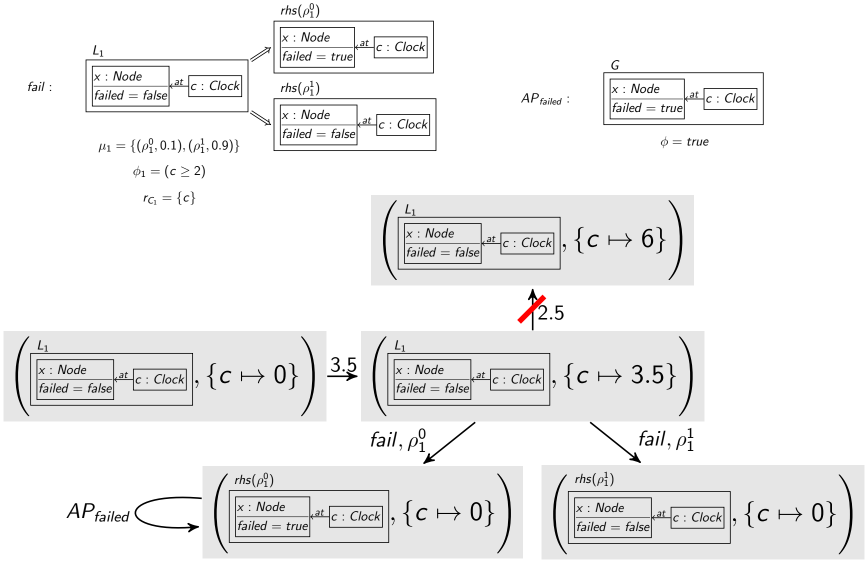

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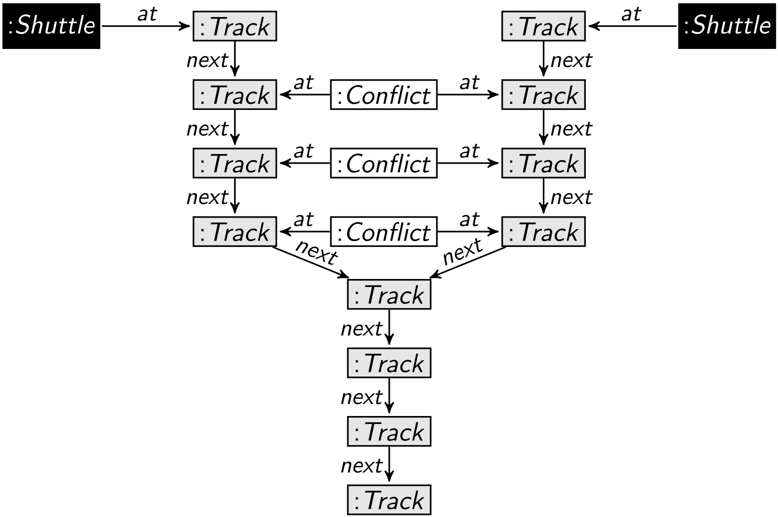

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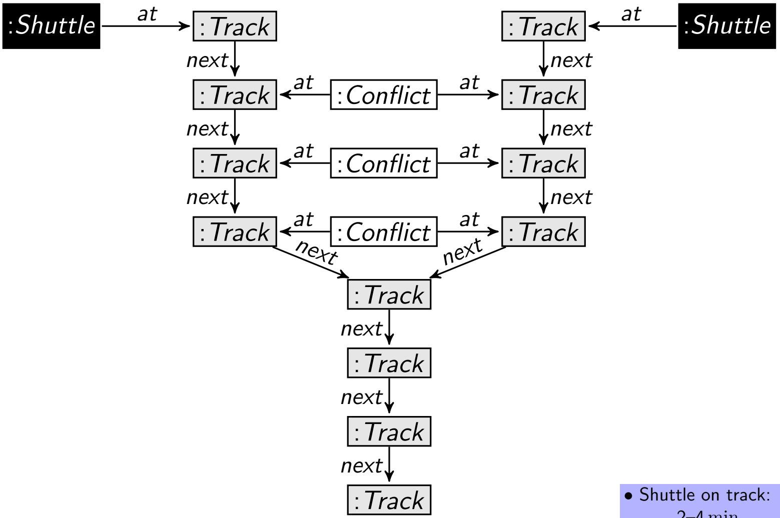

- Shuttle on track:
  2–4 min

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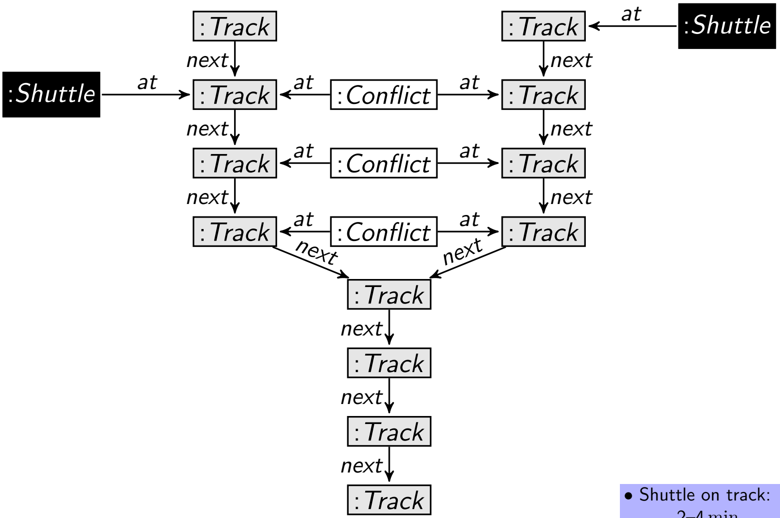

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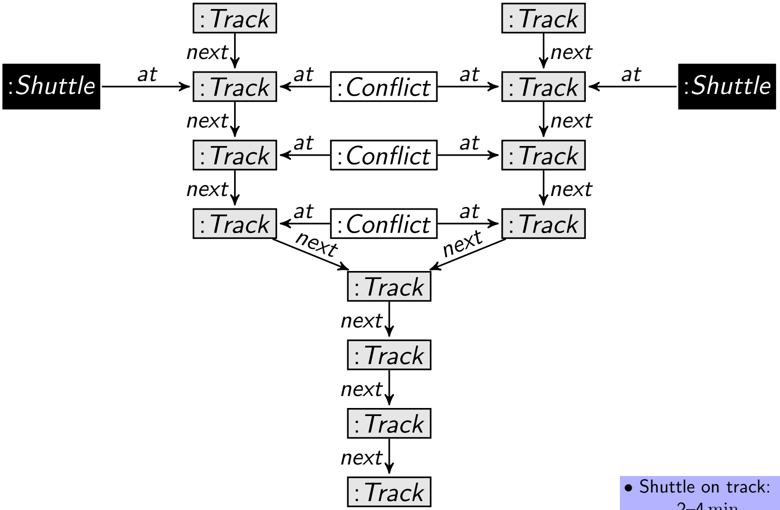

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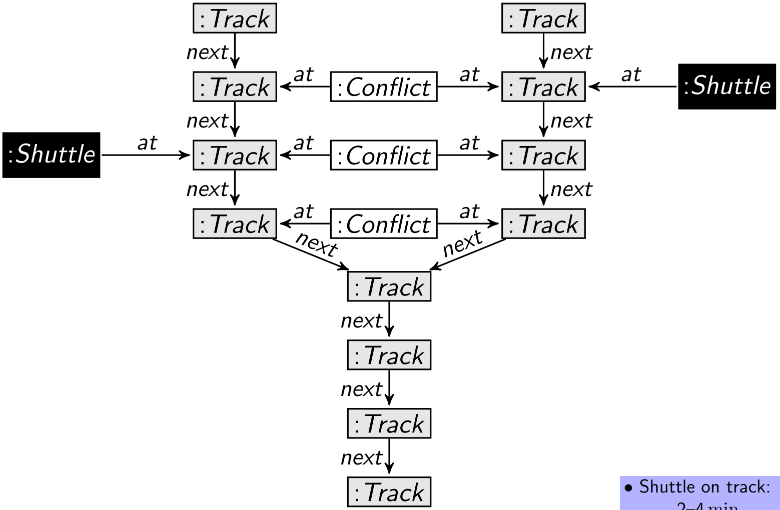

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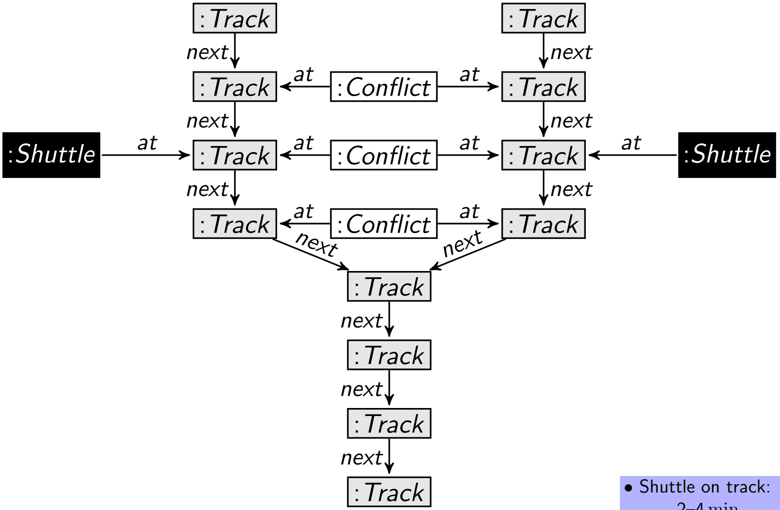

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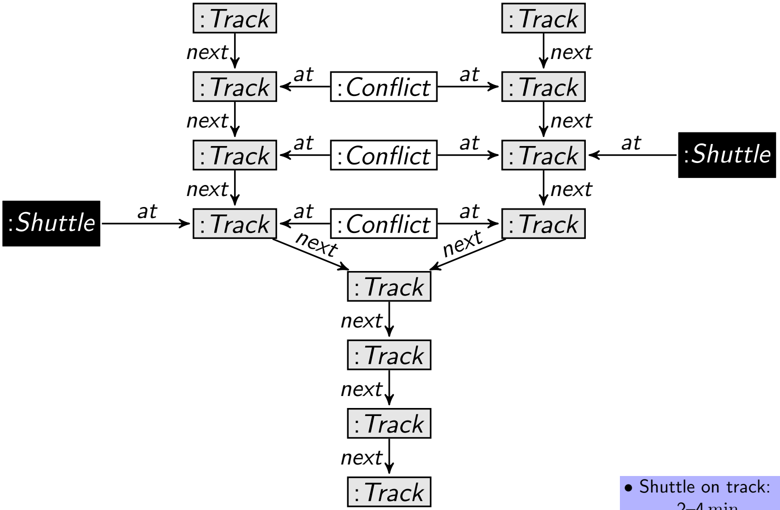

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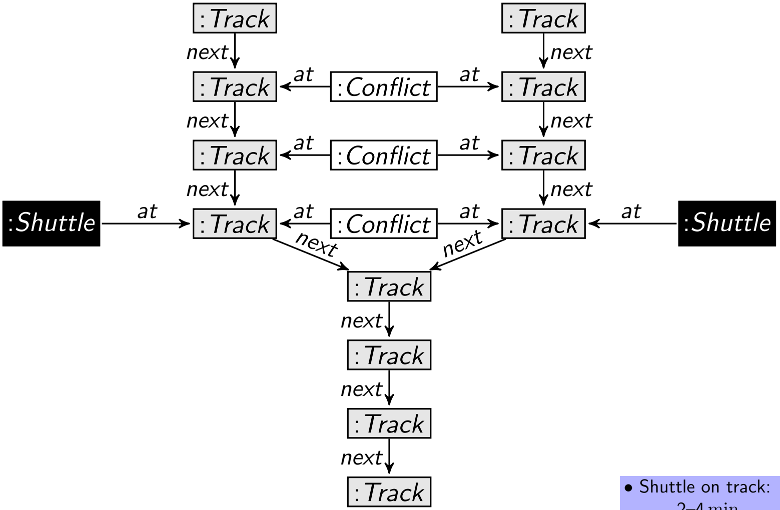

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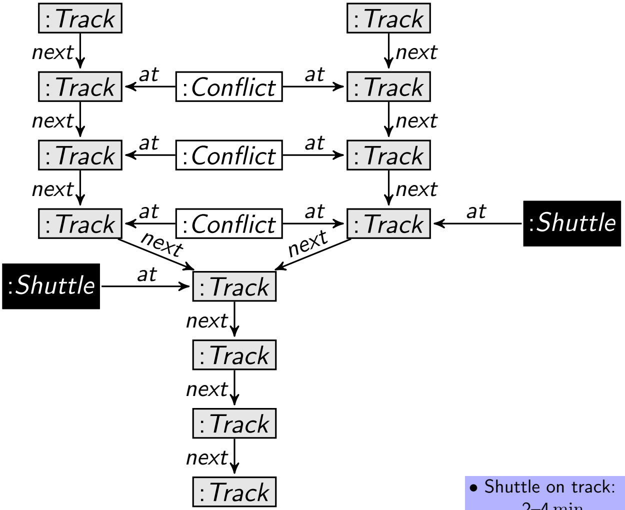

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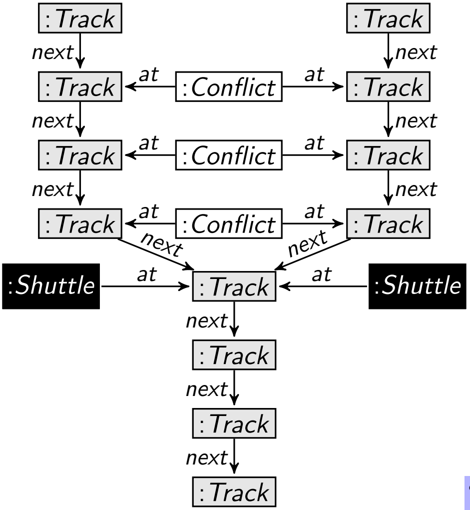

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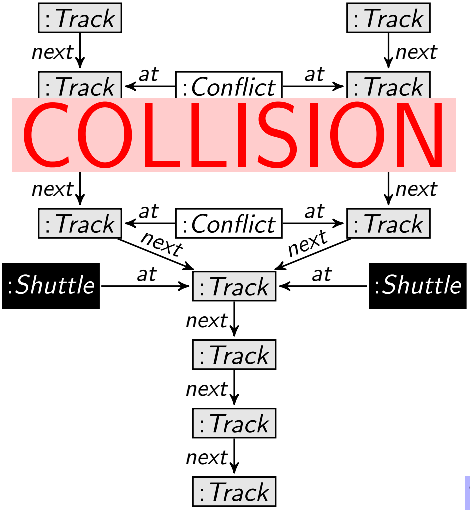

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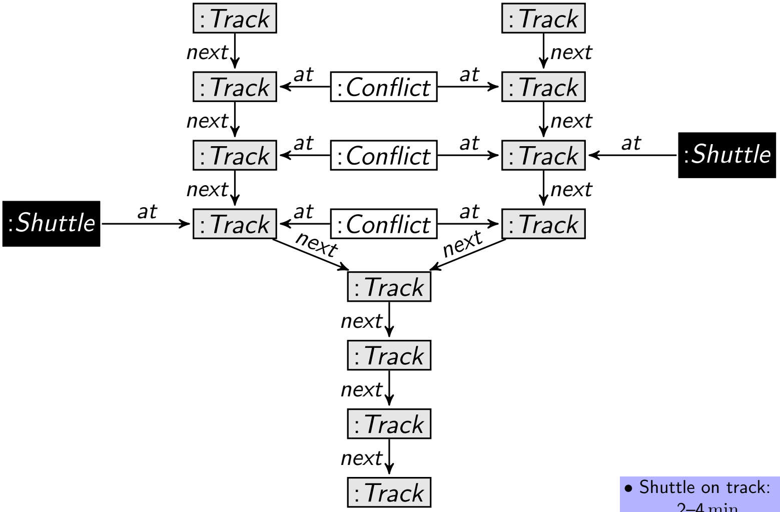

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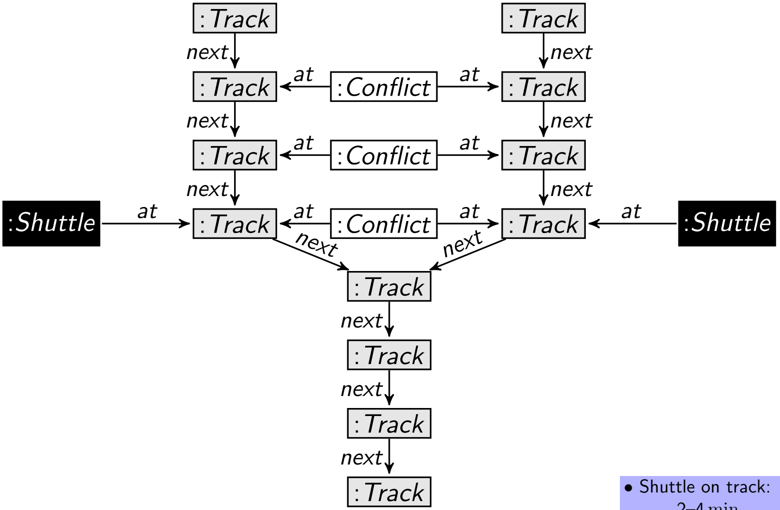

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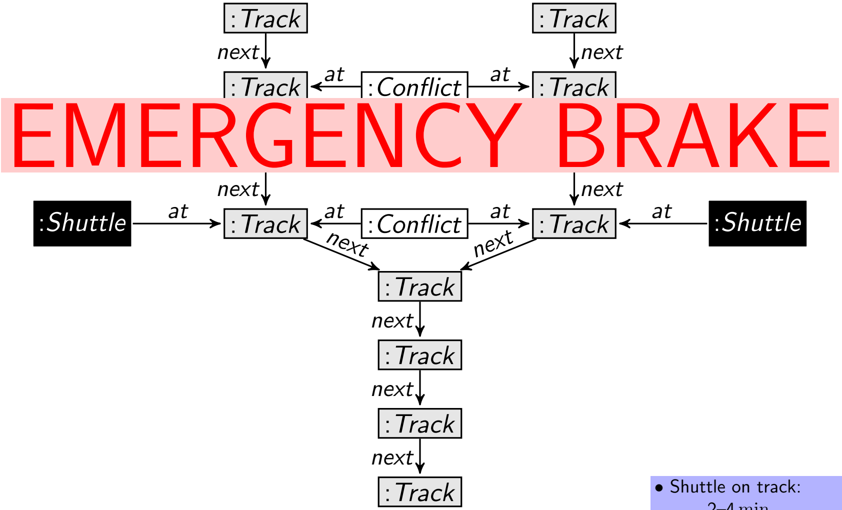

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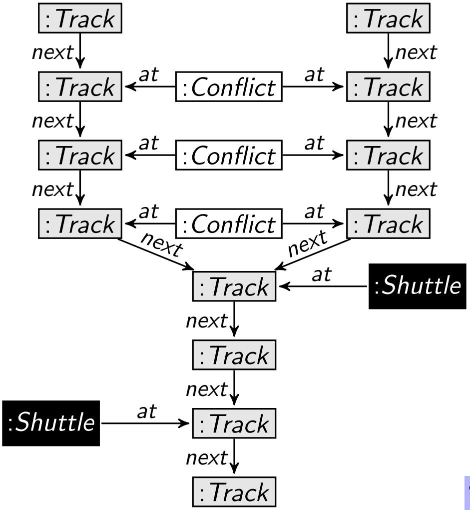

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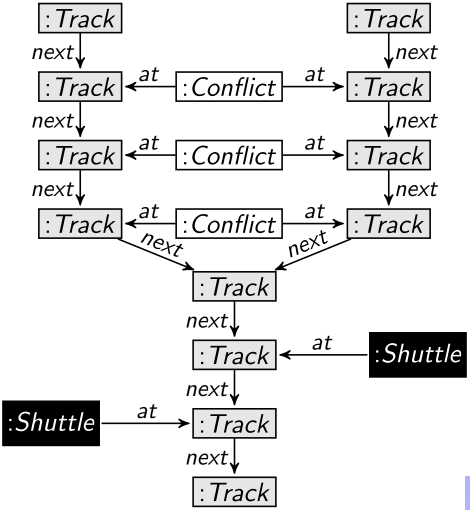

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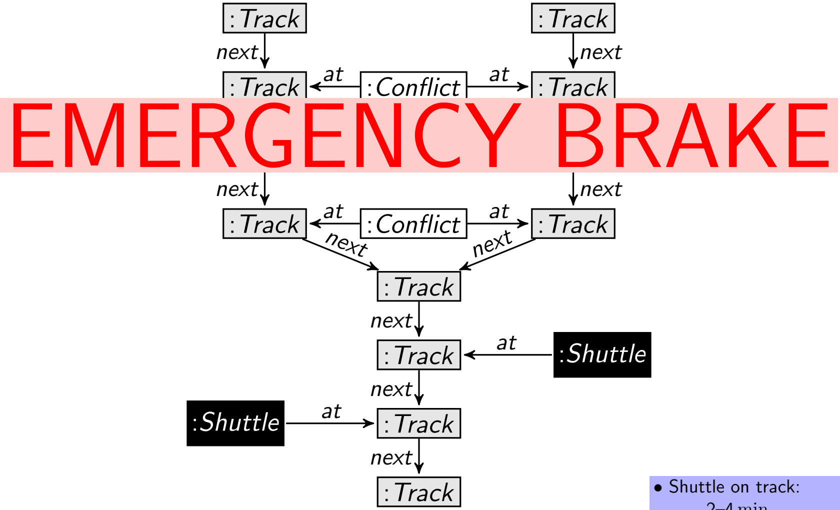

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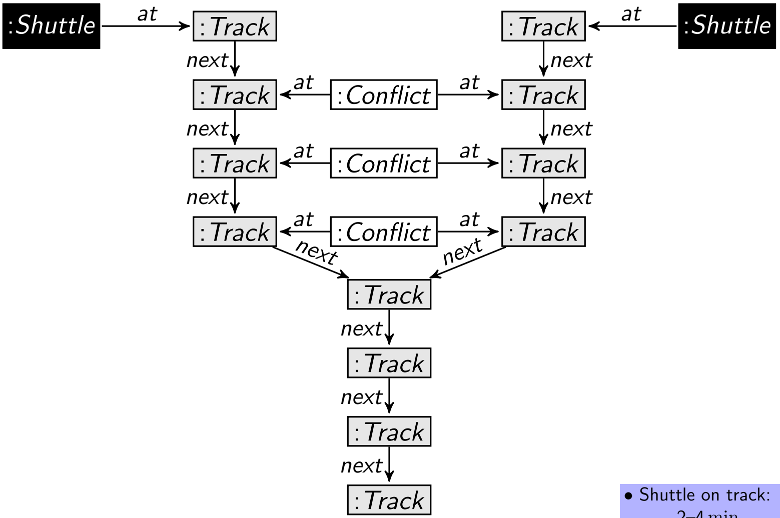

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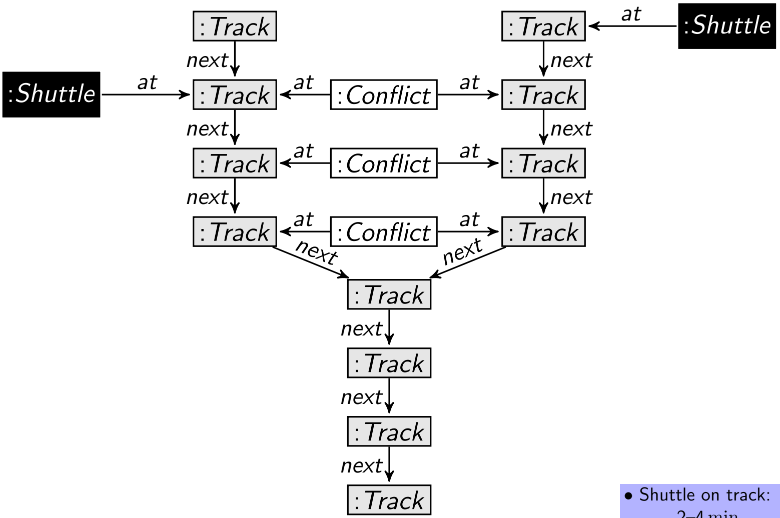

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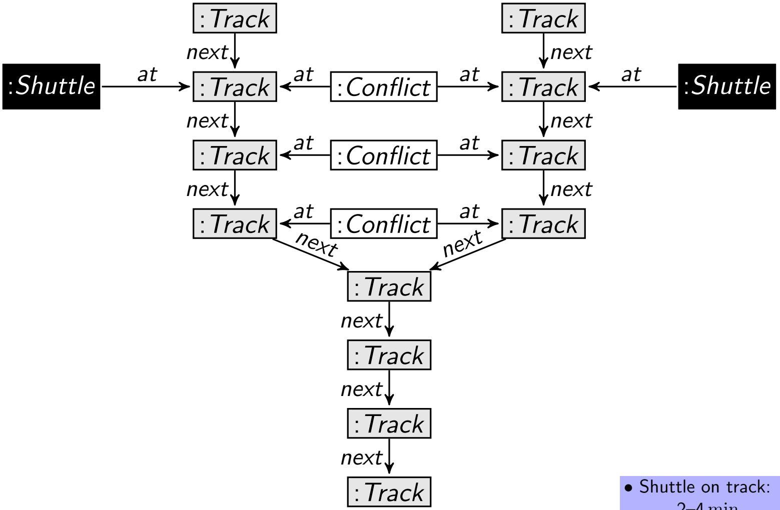

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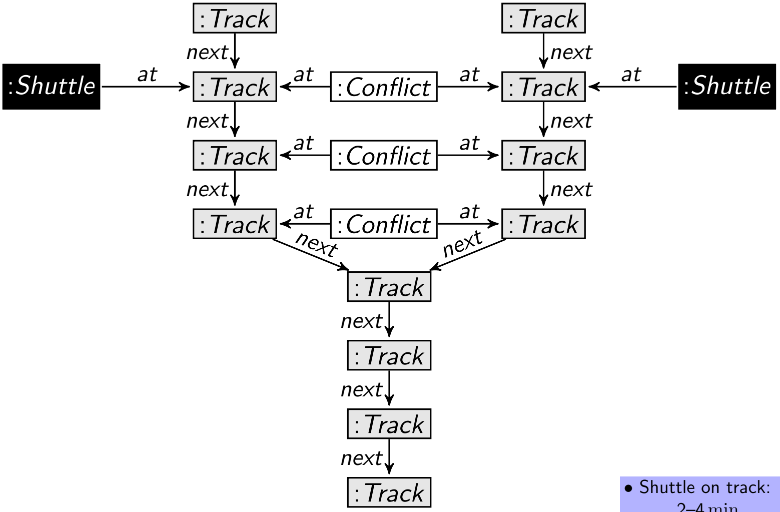

- Shuttle on track:
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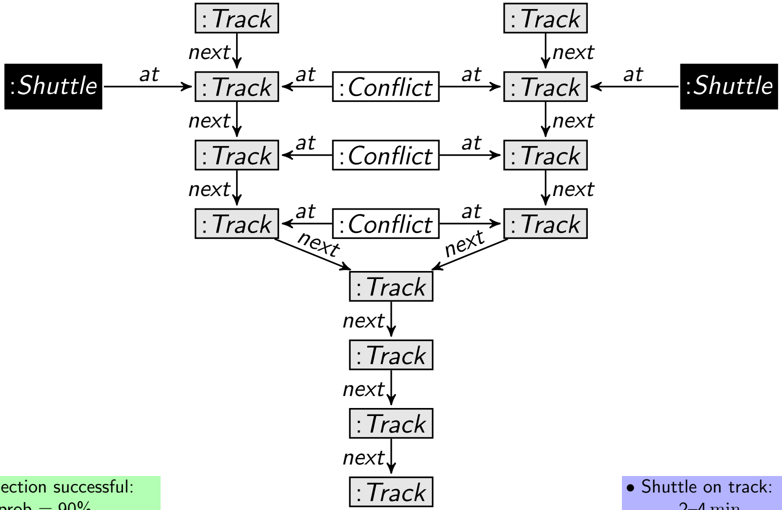
- Connection duration:
  2 min

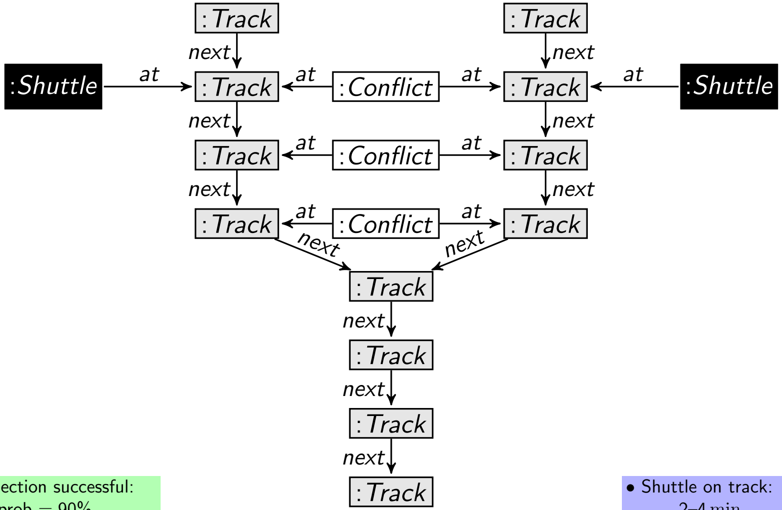
Shuttle Scenario



- Connection successful:
  prob = 90%

- Shuttle on track:
  2–4 min
- Connection duration:
  2 min

Shuttle Scenario



- Connection successful: prob = 90%
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- Shuttle on track: 2–4 min
- Connection duration: 2 min

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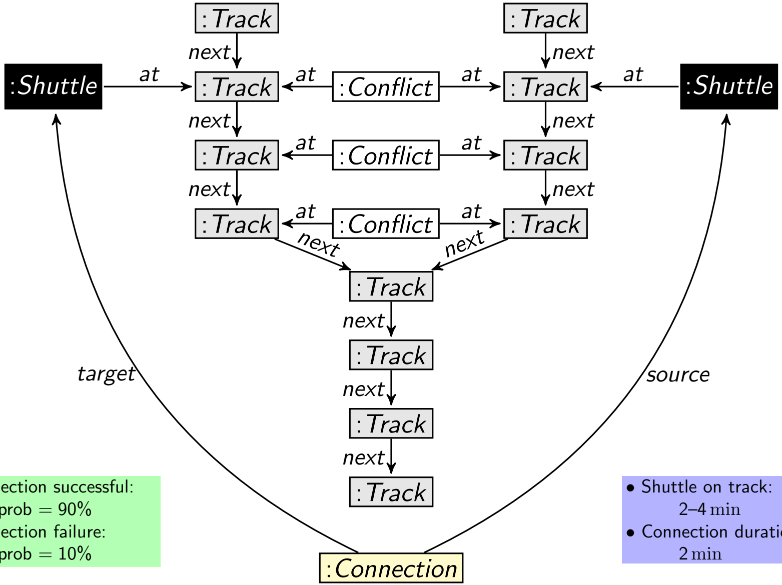

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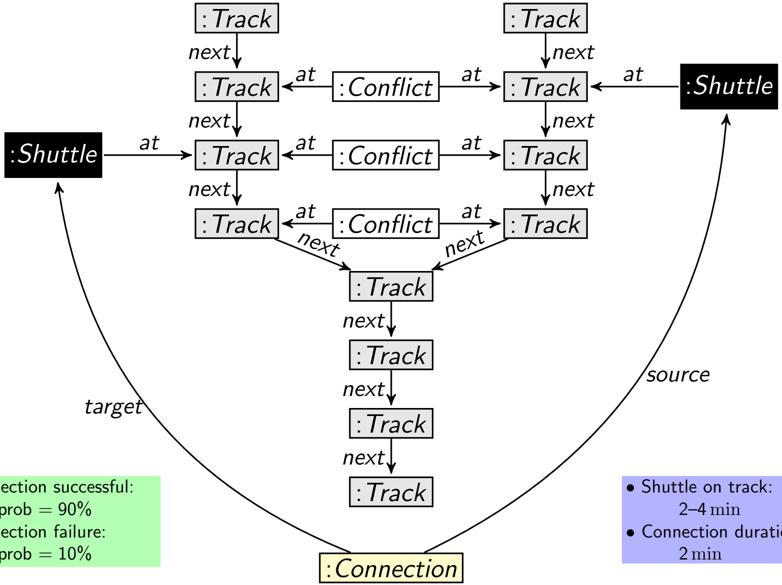

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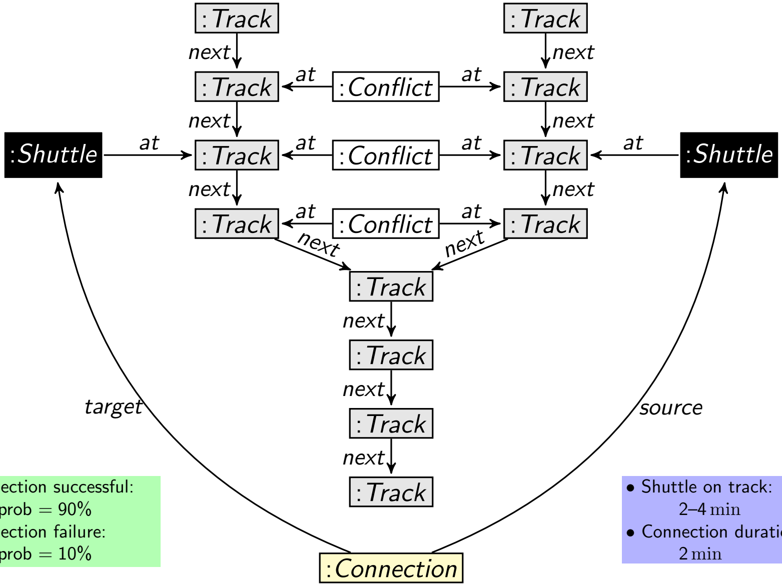

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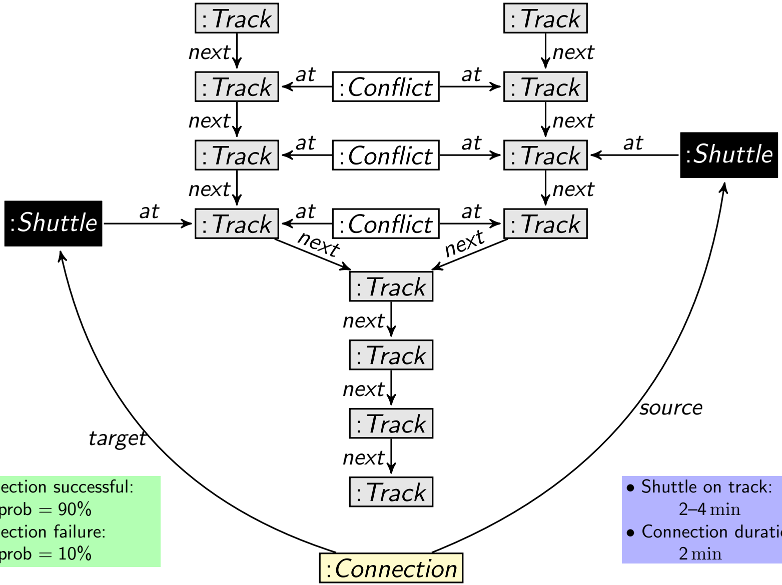

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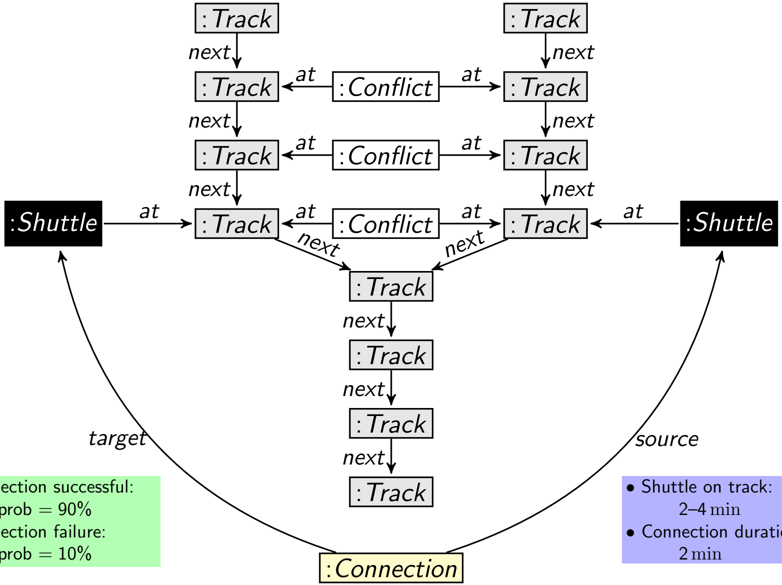

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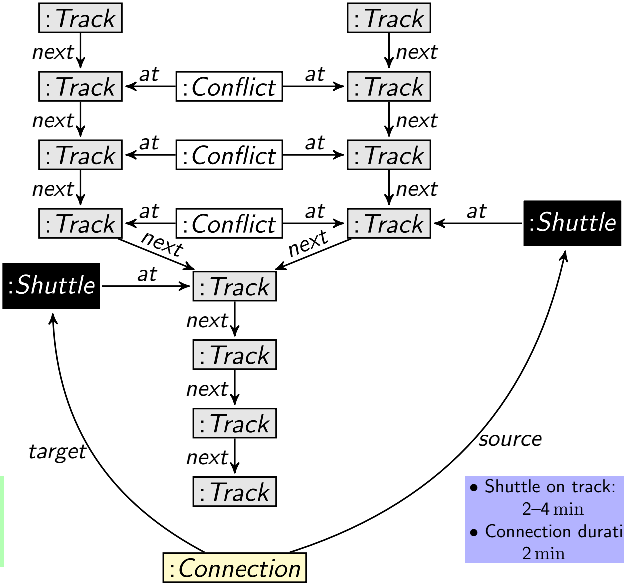

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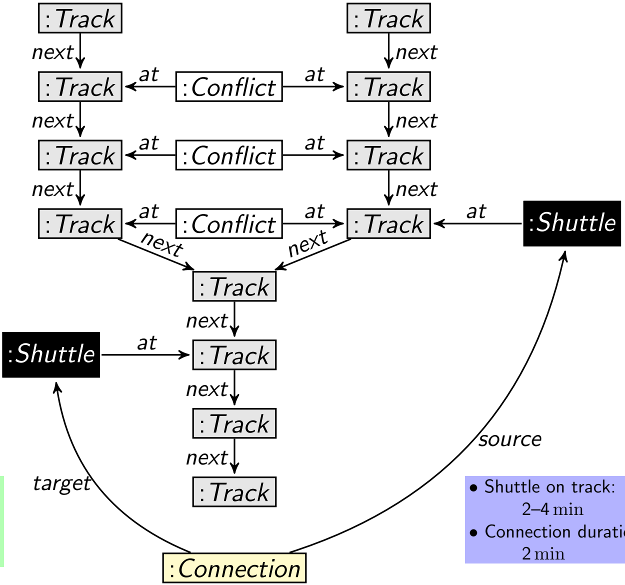

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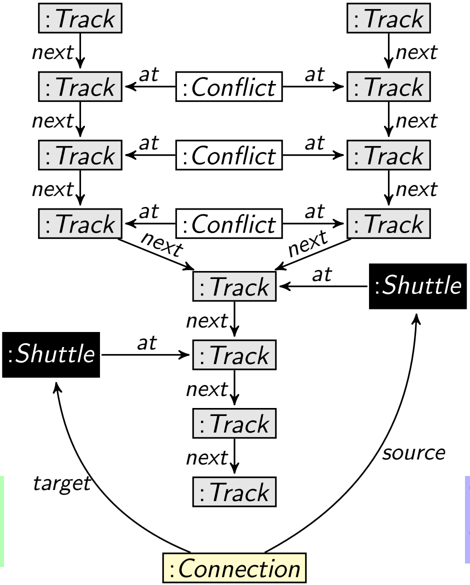

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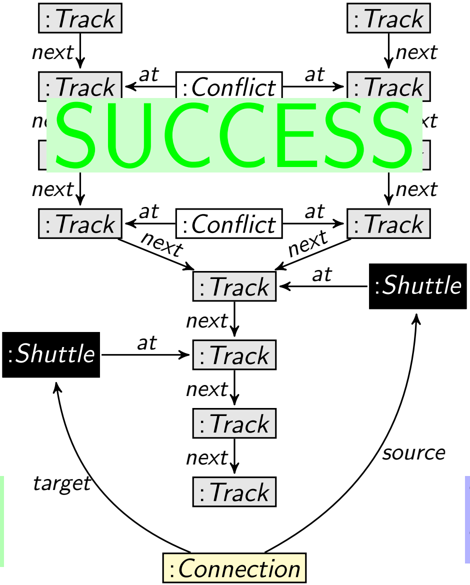

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Experiment 1 (Possibility of Collision)

Can the considered shuttle scenario under any circumstances exhibit a collision?

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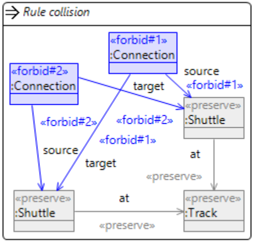
true



Atomic proposition *collision*

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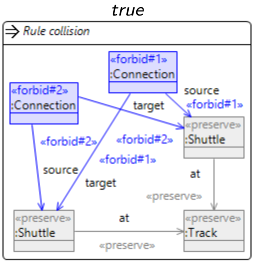

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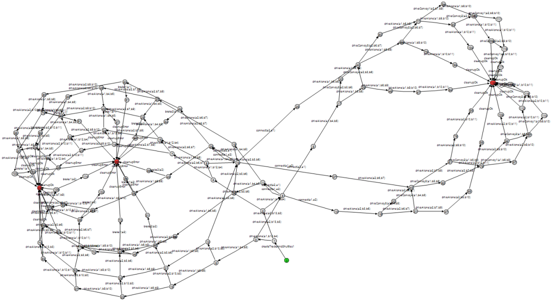

State space for the topology with 3 conflict nodes

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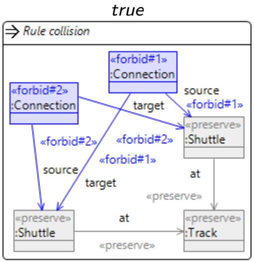

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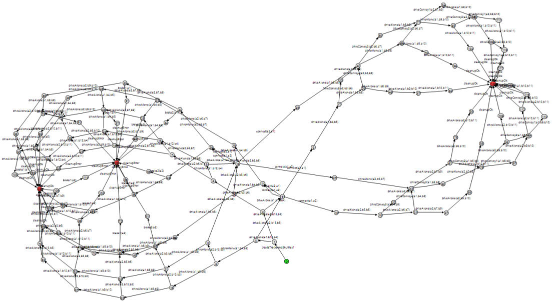

State space for the topology with 3 conflict nodes

Result: No collisions are possible.

Experiment 2 (Probability for Emergency Brake)

What is the maximal probability that a shuttle executes an emergency brake?

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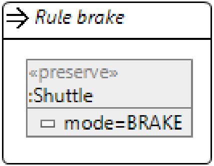
Experiment 2 (Probability for Emergency Brake)

What is the maximal probability that a shuttle executes an emergency brake?

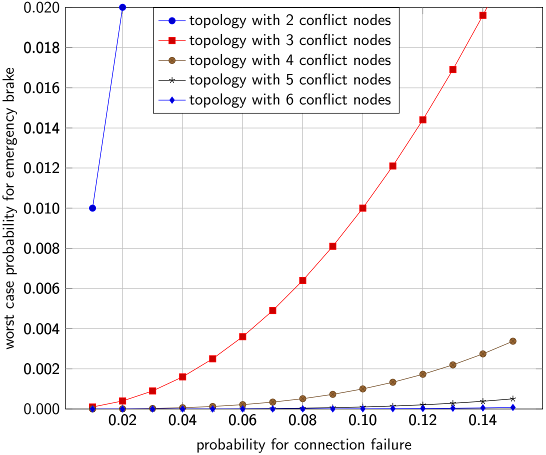
- State spaces for topologies with 2–6 conflict nodes generated by HENSHIN
- State spaces exported to PRISM
- Property for braking behavior in PRISM notation: $Pmax = ?\,[F\ „brake“]$

true

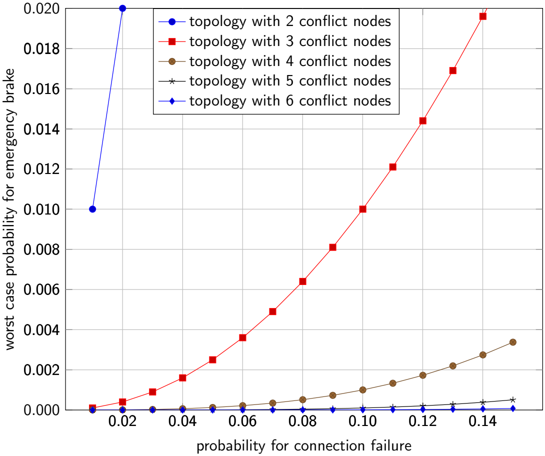


Atomic proposition *brake*

Experiment 2 (Probability for Emergency Brake)



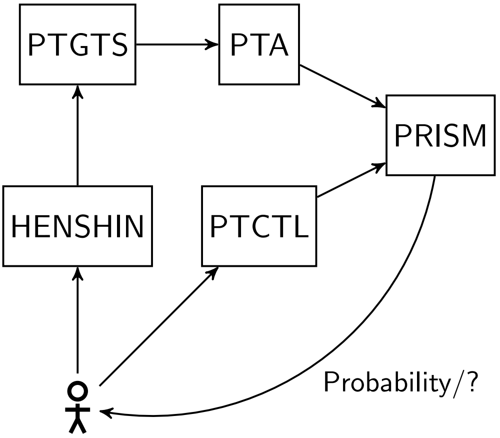
Experiment 2 (Probability for Emergency Brake)



Result: For a desired worst case probability (y-axis) and a communication service quality (x-axis) we can determine the minimal required number of conflict nodes.

Table of Contents

Conclusion



Future Work

- Extension of PTGTSs to interval probabilities (and therefore to IPTA)
- Extension of PTCTL to path properties to specify structure dynamics
- Extension of PRISM to verify path properties of PTGTSs

Thank You For Your Attention