Data-Driven Demand Learning and Dynamic Pricing Strategies in Competitive Markets

Demand Estimation

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Hasso Plattner Institute (EPIC)

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Outline

• Goals of today’s meeting: Demand Estimation

• How to estimate sales probabilities: Simple approaches

• Recommended Exercise: Logistic Regression
**Customer Choice: Buying Books on Amazon**

<table>
<thead>
<tr>
<th>Preis + Versand (inkl. WST)</th>
<th>Zustand</th>
<th>Verkäufer-Information</th>
<th>Lieferung</th>
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</thead>
<tbody>
<tr>
<td>EUR 44,90</td>
<td>Gebraucht - Akzeptabel</td>
<td>ialvamani ★★★★★ 100% positiv, (4 alle Bewertungen)</td>
<td>Rückfragen <em>Ankunft zwischen April 26 - Mai 2.</em></td>
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<td></td>
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<tr>
<td>EUR 45,00</td>
<td>Gebraucht - Sehr gut</td>
<td>lange_und_springer_antiquariat ★★★★★ 98% positiv in den letzten 12 Monaten. (28,564 Bewertungen insgesamt)</td>
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<tr>
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<td>Gebraucht - Wie neu</td>
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<td>EUR 79,56</td>
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</table>

*Optimieren durch Alles Kochen*

*Versand*
- ✔ Prime
- Versandkostenfrei

*Zustand*
- Neu
- ☐ Gebraucht
- ☐ Wie neu
- ☐ Sehr gut
- ☐ Gut
- ☐ Akzeptabel

*Details:
- Publisher: Springer
- Date of Publication: 2014
- Binding: hard...
# Customer Behavior

<table>
<thead>
<tr>
<th>$k$</th>
<th>Price ($p_k$)</th>
<th>Quality ($q_k$)</th>
<th>Rating ($r_k$)</th>
<th>Feedback ($f_k$)</th>
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<td>100%</td>
<td>4</td>
<td>5 Tage</td>
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<tr>
<td>2</td>
<td>45.00</td>
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<td>98%</td>
<td>28,584</td>
<td>6 Tage</td>
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<tr>
<td>3</td>
<td>65.60</td>
<td>wie neu</td>
<td>89%</td>
<td>439</td>
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<td>90%</td>
<td>338</td>
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<td>...</td>
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<td>...</td>
</tr>
<tr>
<td>$K$</td>
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### A Seller’s Perspective: Observable Data

<table>
<thead>
<tr>
<th>period</th>
<th>sale</th>
<th>price</th>
<th>rank</th>
<th>competitor’s prices for product $i$ (ISBN)</th>
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<tbody>
<tr>
<td>$t$</td>
<td>$y_t^{(i)}$</td>
<td>$a_t^{(i)}$</td>
<td>$r_t^{(i)}$</td>
<td>$p_t^{(i)}$</td>
</tr>
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<td>19</td>
<td>3</td>
<td>13</td>
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<tr>
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<td>0</td>
<td>15</td>
<td>2</td>
<td>13</td>
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<tr>
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<td>10</td>
<td>1</td>
<td>13</td>
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<tr>
<td>5</td>
<td>1</td>
<td>12</td>
<td>2</td>
<td>11</td>
</tr>
<tr>
<td>6</td>
<td>0</td>
<td>15</td>
<td>3</td>
<td>11</td>
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Goal

• We have: Market data + Sales data

• We want: Optimize prices + Maximize expected profits

• We need: Sales probabilities for our offer prices

• We use: Regression models, e.g, Logistic regression
Approach: Maximum Likelihood Estimation

• Idea: (1) Choose a model + (2) Find the best calibration

• Example: Coin Toss

• Data: 010111010100010001010010001100000

• Model: Bernoulli Experiment with success probability $p$

• Calibration: Which model, i.e., which $p$ explains our data best?
Our Model: Bernoulli Distribution

- Random variable $Y$ sale occurred (1 yes, 0 no)
- Success probability $P(Y = 1) = p$ and $P(Y = 0) = 1 - p$
- Bernoulli distribution $P(Y = k) = p^k \cdot (1 - p)^{1-k}, \quad k = 0,1$
- (Binomial distribution) $P(Y = k) = \binom{n}{k} \cdot p^k \cdot (1 - p)^{n-k}, \quad k = 0,\ldots,n \quad (n=1)$
Likelihood Function

- Bernoulli distribution \( P(Y = k) = p^k \cdot (1 - p)^{1-k}, \ k = 0,1 \)

- Consider observed data \( \bar{y} = (y_1, \ldots, y_N), \ y_i \in \{0,1\}, \ i = 1,\ldots, N \)

- Probability for our data \( P(Y_i = y_i) = p^{y_i} \cdot (1 - p)^{1-y_i}, \ y_i \in \{0,1\} \)

- Joint probability \( P(Y_1 = y_1, \ldots, Y_N = y_N) = \prod_{i=1}^{N} P(Y_i = y_i) = \prod_{i=1}^{N} p^{y_i} \cdot (1 - p)^{1-y_i} \)

(Likelihood Function)

- Now, maximize the joint probability over the success probability \( p \)!
Maximize the Likelihood Function

- \( \max P(Y_1 = y_1, ..., Y_N = y_N) \) \quad i.i.d.: independent, identically distributed

- \( \max_p \prod_{i=1}^{N} P(Y_i = y_i) \)

- \( \max_{p \in [0,1]} \prod_{i=1}^{N} p^{y_i} \cdot (1 - p)^{1-y_i} \)

Actually, we wanted to find the best \( p \).

- \( \arg \max_{p \in [0,1]} \prod_{i=1}^{N} p^{y_i} \cdot (1 - p)^{1-y_i} \)

We are interested in First Order Conditions. Hence, we do not like products!
Monotone Increasing Transformations

\[
\arg\max_{p \in [0,1]} \left\{ \prod_{i=1}^{N} p^{y_i} \cdot (1 - p)^{1-y_i} \right\}
\]

= \arg\max_{p \in [0,1]} \left\{ 5 \cdot \left( \prod_{i=1}^{N} p^{y_i} \cdot (1 - p)^{1-y_i} \right) + 17 \right\} \quad ? \quad \text{(linear)}

= \arg\max_{p \in [0,1]} \left\{ \left( \prod_{i=1}^{N} p^{y_i} \cdot (1 - p)^{1-y_i} \right)^2 \right\} \quad ?? \quad \text{(convex)}

= \arg\max_{p \in [0,1]} \left\{ \ln \left( \prod_{i=1}^{N} p^{y_i} \cdot (1 - p)^{1-y_i} \right) \right\} \quad ??\quad \text{(concave)}
Log-Likelihood Function

\[
\arg\max_p P(Y_1 = y_1, \ldots, Y_N = y_N) \\
= \arg\max_{p \in [0,1]} \left\{ \ln \left( \prod_{i=1}^{N} p^{y_i} \cdot (1 - p)^{1-y_i} \right) \right\} \\
= \arg\max_{p \in [0,1]} \left\{ \sum_{i=1}^{N} \ln \left( p^{y_i} \cdot (1 - p)^{1-y_i} \right) \right\} \\
= \arg\max_{p \in [0,1]} \left\{ \sum_{i=1}^{N} \left( \ln \left( p^{y_i} \right) + \ln \left( (1 - p)^{1-y_i} \right) \right) \right\} \\
= \arg\max_{p \in [0,1]} \left\{ \sum_{i=1}^{N} \left( y_i \cdot \ln(p) + (1 - y_i) \cdot \ln(1 - p) \right) \right\}
\]
Optimization

- FOC: \( \frac{\partial}{\partial p} P(Y_1 = y_1, \ldots, Y_N = y_N) = 0 \)

\[
\sum_{i=1}^{N} \left( y_i \cdot \ln(p)' + (1 - y_i) \cdot \ln(1 - p)' \right) = 0
\]

- Solve for \( p \).

- 1 Variable, 1 Equation  (Unique solution \( p^* \))

- **Result**: Our data fits to the model \( P(Y = 1) = p^* \) and \( P(Y = 0) = 1 - p^* \).
Generalization: Demand Estimation On Amazon

- Regular price adjustments (e.g., time intervals of ca. 2 hours)

- Observation of market conditions (at the time of price adjustments)
  
  e.g., Competitors’ Prices, Quality, Rating, Shipping Time, etc.

- Sales observations: Points in time

- Rare events, i.e., 0 or 1 sales between price adjustments (2 hours)
## A Seller’s Data Set

<table>
<thead>
<tr>
<th>period</th>
<th>sale $y_t^{(i)}$</th>
<th>price $a_t^{(i)}$</th>
<th>rank $r_t^{(i)}$</th>
<th>competitor’s prices for product $i$ (ISBN)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t$</td>
<td></td>
<td></td>
<td></td>
<td>$p_{t,1}^{(i)}$</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>19</td>
<td>3</td>
<td>13</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>15</td>
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Estimation of Sales Probabilities

• Goal: Quantify sales probabilities as function of our offer price

• Idea: Sales probabilities should depend on market conditions

• Approach: Maximum Likelihood

(1) Choose family of models: Logistic function

(2) Define explanatory variables (based on our data)

(3) Calibrate model: Find model coefficients

(4) Result: Quantify sales probabilities for any market situation!
Explanatory Variables

- Data: Market situation in $t$: $\bar{s} = (t, p_1, ..., p_K, q_1, ..., p_K, r_1, ..., r_K, f_1, ..., f_K, ...)$

- Define explanatory variables (What could affect decisions?):
  
  $x_1(a, \bar{s}) := 1$  
  (Intercept)

  $x_2(a, \bar{s}) := \text{price rank}$  
  (Rank of offer price within competitors’ prices)

  $x_3(a, \bar{s}) := a - \min_{k=1,...,K} p_k$  
  (Price difference to best competitor)

  $x_4(a, \bar{s}) := \text{quality rank}$  
  (Rank of our product condition)

  $x_5(a, \bar{s}) := \#\text{commercials}$  
  (Number of competitors with feedback > 10000)

  $x_6(a, \bar{s}) := \text{combinations}$  
  (Number of comp. with better price + better quality)

  $x_7(a, \bar{s}) := 1_{\{a \mod 10 = 9\}}$  
  (Psychological Prices)

  ...
One Family of Models: Logistic Function

- \( P(Y = 1 | \tilde{x}(a, \tilde{s})) := \frac{e^{\tilde{x} \tilde{\beta}}}{1 + e^{\tilde{x} \tilde{\beta}}} \)

\[
\frac{\exp(\beta_1 \cdot x_1(a, \tilde{s}) + \beta_2 \cdot x_2(a, \tilde{s}) + \ldots)}{1 + \exp(\beta_1 \cdot x_1(a, \tilde{s}) + \beta_2 \cdot x_2(a, \tilde{s}) + \ldots)} \in (0, 1)
\]

- There are other families, but this is a good family

- Maximum Likelihood Estimation:

Find best \( \tilde{\beta} \) coefficients for our data \( y_t, \tilde{x}(a_t, \tilde{s}_t), t = 1, \ldots, N \)
Maximize the Log-Likelihood Function

- Recall:

\[
\arg \max_p P(Y_1 = y_1, ..., Y_N = y_N) = \arg \max_{p \in [0, 1]} \left\{ \sum_{i=1}^N \left( y_i \cdot \ln(p) + (1 - y_i) \cdot \ln(1 - p) \right) \right\}
\]

- Now, we have the conditional probabilities:

\[
\arg \max_{\tilde{\beta}} P(Y_1 = y_1 \mid a_1, \tilde{s}_1, \ldots, Y_N = y_N \mid a_N, \tilde{s}_N)
\]

\[
= \arg \max_{\beta_m \in \mathbb{R}, m = 1, \ldots, M} \left\{ \sum_{i=1}^N \left( y_i \cdot \ln \left( \frac{e^{\tilde{x}(a_i, \tilde{s}_i)\tilde{\beta}}}{1 + e^{\tilde{x}(a_i, \tilde{s}_i)\tilde{\beta}}} \right) + (1 - y_i) \cdot \ln \left( \frac{1 - e^{\tilde{x}(a_i, \tilde{s}_i)\tilde{\beta}}}{1 + e^{\tilde{x}(a_i, \tilde{s}_i)\tilde{\beta}}} \right) \right) \right\}
\]
Optimization

- **FOC:** \[ \frac{\partial}{\partial \beta} P(Y_1 = y_1 \mid a_1, \bar{s}_1, \ldots, Y_N = y_N \mid a_N, \bar{s}_N) = 0 \]

\[ \sum_{i=1}^{N} \left( y_i \cdot \frac{\partial}{\partial \beta_m} \ln \left( \frac{e^{\bar{x}(a_i, \bar{s}_i)' \bar{\beta}}}{1 + e^{\bar{x}(a_i, \bar{s}_i)' \bar{\beta}}} \right) + (1 - y_i) \cdot \frac{\partial}{\partial \beta_m} \ln \left( 1 - \frac{e^{\bar{x}(a_i, \bar{s}_i)' \bar{\beta}}}{1 + e^{\bar{x}(a_i, \bar{s}_i)' \bar{\beta}}} \right) \right) = 0, \ m = 1, \ldots, M \]

- Solve **the system** for \( \bar{\beta} = (\beta_1, \ldots, \beta_M) \)

- **M Variables, M Equations**  (Unique solution \( \bar{\beta}^* = (\beta^*_M, \ldots, \beta^*_M) \))

- **Result:** Our data fits to the model \( P(Y = 1 \mid \bar{x}(a, \bar{s})) := e^{\bar{x}(a, \bar{s})' \bar{\beta}^*} / (1 + e^{\bar{x}(a, \bar{s})' \bar{\beta}^*}) \)
Application of the Model Obtained

- Observe current market situation for a product: $\bar{s}$

- For any admissible offer prices $a$ we can evaluate $\bar{x}(a, \bar{s})$ and obtain

$$P(Y = 1 | \bar{x}(a, \bar{s})) := \frac{e^{\bar{x}(a, \bar{s})' \bar{\beta}^*}}{1 + e^{\bar{x}(a, \bar{s})' \bar{\beta}^*}}$$

- Now, we can optimize expected profits (for one time interval):

$$\max_{a \geq 0} \left\{ (a - c) \cdot \frac{e^{\bar{x}(a, \bar{s})' \bar{\beta}^*}}{1 + e^{\bar{x}(a, \bar{s})' \bar{\beta}^*}} \right\}$$
Prediction of Sales Probabilities

- Example: Competitor’s prices \( \bar{p} = (4.26, 5.18, 5.31, 5.55, 5.86, \ldots) \)

\[
\begin{align*}
\text{sales probability } P(a) & \quad \text{expected profit } (a-3) \cdot P(a)
\end{align*}
\]
Summary

(+) Logistic Regression is simple and robust

(+) Allows for many observations $N$ and many features $M$

(+) Plausibility Checks & Closed Form Expressions

(+/−) Definition of Customized Explanatory Variables

(−) No dependencies between variables

(−) Limited to binary dependent variables
What is a good Model?

- Compare “Goodness of fit” measures

- Logit: $AIC$ (low is good, trade-off between fit and number of variables $M$)

  $AIC = -2 \sum_{i=1}^{N} (y_i \cdot \ln p_i + (1 - y_i) \cdot \ln(1 - p_i)) + 2 \cdot M$

  Note, $p_i$ depends on all features $x_i$ and the optimal $\beta^*$ coefficients.

- Be creative: Test different variables and find the smallest $AIC$ value.

  Hint: Not quantity but quality counts!
Recommended Exercise – Demand Estimation

- Create random market situations with multiple sellers
- Choose a specific Buying Behavior, e.g., Approach II (with 0.6, 0.3, 0.1)
- Simulate sales events for different market situations
- **Estimate and compare** sales probabilities
- Use different combinations of explanatory variables
- Compare the Goodness of Fit of the different models
Recommended Exercise – Demand Estimation

- Our prices $p_i \sim U(1, 20), \ i = 1, \ldots, N$, for $N$ market situations

- Competitors’ prices $p_{i,k}^{(c)} \sim U(1, 20), \ k = 1, \ldots, 5$, for 5 competitors

- Observed sales $y_i \sim (=1 \text{ with } 60\%, \ 30\%, \ 10\%)$ if $\text{rank}(p_i) = 1, 2, \text{ or } 3$

- Estimation of sales probabilities $P(p_i, \tilde{p}_i^{(c)}; \tilde{\beta}^*)$ via logit model

- Explanatory variables: intercept, our price, price rank, etc.

- Compute AIC & compare $P(p_i, \tilde{p}_i^{(c)}; \tilde{\beta}^*)$ with (60%, 30%, 10%, 0%, 0%, 0%)
Overview

<table>
<thead>
<tr>
<th>Date</th>
<th>Event</th>
</tr>
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<tbody>
<tr>
<td>April 25</td>
<td>Customer Behavior</td>
</tr>
<tr>
<td>May 2</td>
<td>Demand Estimation</td>
</tr>
<tr>
<td>May 9</td>
<td>Pricing Strategies I</td>
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<td>May 16</td>
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<tr>
<td>May 23</td>
<td>Pricing Strategies II</td>
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<td>May 30</td>
<td>Dynamic Pricing Challenge &amp; Price Wars Platform</td>
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<td>June 6</td>
<td>Workshop / Group Meetings</td>
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<td>Presentations (First Results)</td>
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<td>June 20</td>
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<td>July 4</td>
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<td>July 11</td>
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<tr>
<td>July 18</td>
<td>Presentations (Final Results), Feedback, Documentation (Aug/Sep)</td>
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