# Data-Driven Demand Learning and 

# Dynamic Pricing Strategies in Competitive Markets 

## Pricing Strategies

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May 9, 2017

## Outline

- Goals of today's meeting: Pricing Strategies
- How to set offer prices:

Simple Approaches

- Summarizing Exercise

From data to Pricing

## Pricing Competition



A Course in In－Memory Data Management：The Inner Mechanics of In－Memory Databases（Gebundene Ausgabe） von Hasso Plattner（Autor）
Schreiben Sie die erste Bewertung

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## Observable Data: Market Situations

| seller | price | quality | rating | feedback | shipping |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $k$ | $p_{k}$ | $q_{k}$ | $r_{k}$ | $f_{k}$ | $c_{k}$ |
| 1 | 44.90 | akzeptabel | $100 \%$ | 4 | 5 Tage |
| $\mathbf{2}$ | $\mathbf{4 5 . 0 0}$ | sehr gut | $\mathbf{9 8 \%}$ | $\mathbf{2 8 , 5 8 4}$ | 6 Tage |
| 3 | 65.60 | wie neu | $89 \%$ | 439 | 11 Tage |
| 4 | 79.56 | sehr gut | $90 \%$ | 338 | 10 Tage |
| $\ldots$ |  |  |  |  |  |
| $K$ |  |  |  |  |  |

## Price Adjustments in Market Situations

| time | adjusted price | rank | market situation |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $t$ | $a_{t}$ | $r_{t}$ | $p_{t, 1}$ | $p_{t, 2}$ | $p_{t, 3}$ | $p_{t, 4}$ | $\ldots p_{t, K}$ |  |
| 0 | $\mathbf{1 9}$ | 3 | 13 | 17 | 20 | 25 |  |  |
| 1 | $\mathbf{1 6}$ | 2 | 13 | 17 | 20 | 25 |  |  |
| 2 | $\mathbf{1 2}$ | 1 | 13 | 15 | 20 | $/$ |  |  |
| 3 | $\mathbf{1 0}$ | 1 | 11 | 15 | 20 | 22 |  |  |
| 4 | $\mathbf{1 4}$ | 2 | 11 | 15 | 20 | 24 |  |  |
| 5 | $\mathbf{1 9}$ | 3 | 11 | 13 | 20 | 24 |  |  |
| $\ldots$ |  |  |  |  |  |  |  |  |

## Observable Data: Sales within Adjustment Periods

| period | sale | price | rank | market situation |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $(t, t+1)$ | $y_{t}^{(1)}$ | $a_{t}$ | $r_{t}$ | $p_{t, 1}$ | $p_{t, 2}$ | $p_{t, 3}$ | $p_{t, 4}$ | $\ldots$ |$p_{t, K}$

Extension: Multiple Sales

| period | sales | price | rank | competitor's prices |  |  |  |  |  | for product $i$ (ISBN) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $(t, t+1)$ | $y_{t}^{(1)}$ | $a_{t}$ | $r_{t}$ | $p_{t, 1}$ | $p_{t, 2}$ | $p_{t, 3}$ | $p_{t, 4}$ | $\ldots p_{t, K}$ |  |  |
| $(0,1)$ | $\mathbf{3}$ | $\mathbf{1 9}$ | 3 | 13 | 17 | 20 | 25 |  |  |  |
| $(1,2)$ | $\mathbf{1 5}$ | $\mathbf{1 6}$ | 2 | 13 | 17 | 20 | 25 |  |  |  |
| $(2,3)$ | $\mathbf{2 3}$ | $\mathbf{1 2}$ | 1 | 13 | 15 | 20 | $/$ |  |  |  |
| $(3,4)$ | $\mathbf{1 9}$ | $\mathbf{1 0}$ | 1 | 13 | 15 | 20 | 22 |  |  |  |
| $(4,5)$ | $\mathbf{2 1}$ | $\mathbf{1 4}$ | 2 | 11 | 15 | 20 | 24 |  |  |  |
| $(5,6)$ | $\mathbf{9}$ | $\mathbf{1 9}$ | 3 | 11 | 13 | 20 | 24 |  |  |  |
| $\ldots$ |  |  |  |  |  |  |  |  |  |  |

## Simple Approach: Least Squares Regression

- Idea: explain the „dependent variable" by „explanatory variables"
- „Dependent variable": number of sales $y$ (of our firm)
- „Explanatory variables": price rank $r$
price difference to best competitor's price
time (day time, weekday, month etc.)
ratings, shipping time, ...
- Remember: Derive the $\beta^{*}$ - coefficients for every explanatory variable by minimizing sum of squared deviations (over all observations)


## Example: Expected Sales as Function of Price Rank

- Explanatory variable: $x_{i}^{(1)}(a, \vec{p})=1$, price rank $x_{i}^{(2)}(a, \vec{p})=r_{i}(a, \vec{p})$
- Regression result:
- Expected sales:
- Impact analysis:

Intercept $\beta_{1}^{*}$, price rank impact $\beta_{2}^{*}$

$$
\hat{y}(a, \vec{p})=\beta_{1}^{*}+\beta_{2}^{*} \cdot r(a, \vec{p})
$$

Each better rank boosts the expected number of sales by $\beta_{2}^{*}$ units!

- We can estimate expected sales for all prices $a$ and situations $\vec{p}$ !


## Let's be creative: Multi Linear Regression

- Invent multiple explanatory variables from the raw data!
- Use transformed variables, e.g., $x^{(3)}=r^{2}$

$$
x^{(4)}=\ln (r)
$$

- Use and combine multiple features (customer ratings, shipping time, etc.).
- Same Model:

$$
y(a, \vec{p}, \ldots) \approx \sum_{m=1}^{M} \beta_{m} \cdot x^{(m)}(a, \vec{p}, \ldots)=\vec{\beta}^{\prime} \vec{x}=\hat{y}(\vec{\beta}, \vec{x}(a, \vec{s}))
$$

- LS Minimization: $\min _{\beta_{1}, \ldots, \beta_{M} \in \mathbb{R}}\left\{\sum_{i=1}^{N}\left(y_{i}-\hat{y}_{i}\left(\vec{\beta}, \vec{x}_{i}\right)\right)^{2}\right\}$


## What is a Good Model?

- Compare "Goodness of Fit" measures
- OLS:

$$
R^{2} \quad(\text { share of explained variance in } y \text { ) }
$$

- Model fit: $\quad \hat{y}_{i}=\beta_{1}^{*}+\beta_{2}^{*} \cdot x_{i}^{(2)}+\beta_{3}^{*} \cdot x_{i}^{(3)}+\ldots \approx y_{i}$
- New variance: $\quad V_{A R}=\frac{1}{N} \cdot \sum_{i=1}^{N}\left(y_{i}-\hat{y}_{i}\right)^{2} \quad \leq \operatorname{VAR}=\frac{1}{N} \cdot \sum_{i=1}^{N}(y_{i}-\underbrace{\bar{y}}_{1 / N \cdot} \sum_{i=1})^{2}$
- Goodness of fit: $\quad R^{2}:=1-\frac{V A R_{n e w}}{V A R} \in[0,1] \quad$ (large is good)


## From Forecasts to Sales Probabilities

- We have estimations to sell $\hat{y}^{(h)}(a, \vec{s})$ items at price $a$ within a period of length $h$ which starts with situation $\vec{s}$
- We look for a probability distribution $\tilde{P}^{(h)}(i, a, \vec{s})$ to sell $i$ items at price $a$ within a period of length $h$ which starts with situation $\vec{s}$
- Simple Approach: Poisson Probabilities with mean $\hat{y}^{(h)}(a, \vec{s})$

$$
\tilde{P}(i, a, \vec{s})=\tilde{P}(i, a, \vec{p}, \ldots)=\operatorname{Pois}(\hat{y}(\vec{\beta}, \vec{x}(a, \vec{s})))=\frac{\hat{y}^{i}}{i!} \cdot e^{-\hat{y}}, i=0,1,2, \ldots
$$

## Summary: Demand Estimation

- Explain dependent variable $y_{t}^{(1)}$ by customized explanatory variables $\vec{x}_{t}(\vec{s})$
- Various Regression/ML techniques can be used
- Result: Probability $\tilde{P}^{(h)}(i, a, \vec{s})$ to sell $i$ items at price $a$ within a period of length $h$ which starts with situation $\vec{s}$
- Measure the Goodness of fit of your model/result
- Compare your estimated probabilities $\tilde{P}^{(h)}(i, a, \vec{s})$ with true ones $P^{(h)}(i, a, \vec{s})$


## What Do We Have Learned?

- We can model:
- We can analyze:
- We can estimate:
- We can verify the: Quality of our estimations
- We want to:

Customer Choice

Sales data \& market situations

Sales probabilities for time intervals

Compute optimized prices

## Price Reaction Strategies (Rule-Based)

- Idea:
(1) Observe market situation + (2) Adjust price
- Examples: $a(\vec{s})=a^{(1)}(\vec{p}):=\max \left(c, \min _{k=1, \ldots, K} p_{k}-\varepsilon\right)$

$$
\begin{aligned}
& a(\vec{s})=a^{(n)}(\vec{p}):=\max _{a \in A: \operatorname{rank}(a, \vec{p})=n} a \\
& a(\vec{s})=a^{(\text {random })}(\vec{p}):=\text { if } U(0,1)<0.5 \text { then } a^{(1)}(\vec{p}) \text { else } a^{(2)}(\vec{p})
\end{aligned}
$$

$$
a(\vec{s})=a^{(g a s)}(\vec{p}):=\left\{\begin{array}{cc}
a^{(1)}(\vec{p}) & , p^{\min } \leq \min _{k=1, ., K} p_{k} \leq p^{\max } \\
p^{\max } & , \text { else }
\end{array}\right.
$$

## Price Reaction Strategies (Data-Driven)

- Idea: (1) Observe market situation + (2) Adjust price
(3) Use expected sales probabilities
- Use: Probability $\tilde{P}^{(h)}(i, a, \vec{s})$ to sell $i$ items at price $a$ within a period of length $h$ which starts with market situation $\vec{s}$
- Examples: Maximize short-term profit via

$$
a^{(*)}(\vec{s}):=\underset{a \in A}{\arg \max } \sum_{i=0,1, \ldots} i \cdot(a-c) \cdot \tilde{P}^{(h)}(i, a, \vec{s})
$$

## Mandatory Exercise - Combine all Components

(1) Create random market situations with multiple sellers

Choose randomized prices for our firm (exploration phase)
(2) Choose a specific Buying Behavior, e.g., Approach II (with 0.6, 0.3, 0.1)

Simulate our firm's sales for all market situations
(3) Estimate sales probabilities, e.g., Logit model or Poisson via least squares

Use different combinations of explanatory variables

## Mandatory Exercise - Combine all Components

(4) Measure the goodness of fit of your models, i.e.,

Compare original and estimated sales probabilities
(5) Create new random market situations with multiple sellers

Evaluate your estimated sales probabilities for potential offer prices
Compute prices that maximize expected short-term profits
(6) Simulate sales for all new market situations and your optimized prices

Compare realized profit for rule-based strategies \& the optimized prices

## Overview

| 2 | April 25 | Customer Behavior |
| :--- | :--- | :--- |
| 3 | May 2 | Demand Estimation |
| 4 | May 9 | Pricing Strategies I |
| 5 | May 16 | no Meeting |
| $\mathbf{6}$ | May 23 | Pricing Strategies II (Optimal Solution of the Duopoly Game) |
| 7 | May 30 | Dynamic Pricing Challenge \& Price Wars Platform |
| 8 | June 6 | Workshop / Group Meetings |
| 9 | June 13 | Presentations (First Results) |
| 10 | June 20 | Workshop / Group Meetings |
| 11 | June 27 | no Meeting |
| 12 | July 4 | Workshop / Group Meetings |
| 13 | July 11 | Workshop / Group Meetings |
| $\mathbf{1 4}$ | July 18 | Presentations (Final Results), Feedback, Documentation (Aug/Sep) |

