Algorithm Engineering Group
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Reachability Problems on Temporal Graphs

Motivation Many structures in everyday life can be modelled as networks. They range from abstract constructs, such as (online) social networks, to essential infrastructure, such as transportation networks and power grids. Given their importance, rigorous research has been conducted to better understand real-world networks and evaluate their behavior. Over the years, we have realized that more varied and complex models are required to capture the intricacies that govern networks' attributes and one of the most prominent attributes is that networks tend to be dynamic, i.e., they change over time.

We consider the temporal graphs model, a model used to describe networks whose connections change based on a predefined schedule. Consider, for example, the Berlin transportation system, where there is a path from any station to any other station but these paths are realistic representations only if we consider that trains run on certain times. To model this, assume that each edge of our graph contains time labels defining when the edge is available to traverse. Given such a graph, we want to consider reachability problems under constraints, i.e., if my starting point is Griebnitzee S-Bahn train station at 11 pm on Tuesday, can I reach every station in Berlin? Of course, this is just one example of a real-world network where we can study reachability problems on temporal graphs. See Figure 1 for an example of such a temporal graph.

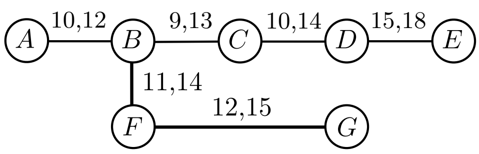

Figure 1: A temporal graph of a network where we assume that every label has traversal time equal to 1. If my starting point is A, I can reach E by taking the edges AB,BC,CD,DE at time steps 12,13,14,18, respectively. I can also reach G by taking the edges AB,BF,FG at time steps 10,14,15, respectively

The goal of this project is to consider reachability problems under constraints on temporal graphs both from a centralized approach and a decentralized one. For the centralized setting, we want to consider network reconfiguration problems where, given a temporal network, we want to identify minimum modifications to the network which allows us to minimize or maximize the reachability sets of each node in the graph. Modifications can be intrusive, such as deleting labels all together, or less invasive, such as delaying or rearranging labels. On the other side, constraints include minimizing the hops needed to reach the other nodes of the network, or minimizing the time of the trip from node u to node v. (See citation [1] for a recent survey on reachability problems on temporal graphs).

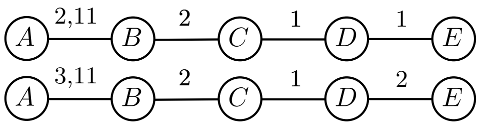

Figure 2: Top: The original temporal graph of a network where we assume that every label has traversal time equal to 1. Note that vertex C reaches vertex A at time step 12 and vertex E is unreachable. Bottom: The modified temporal graph by delaying edges AB and DE. Note that now, vertex C reaches vertex A at time step 4 and vertex E can now be reached at time step 3.

See Figure 2 for an example on how modifications can increase reachability. For the decentralized approach, we will assume that each node of our temporal graph is a selfish agent and can buy edges on the network, in order to satisfy its objective of reaching every other node. We will consider various objectives and multiple host networks.

The main task is twofold: (i) develop and analyze efficient algorithms that modify the network in order to achieve the reachability criteria that we set, (ii) study the existence of equilibria as well as upper and lower bounds on the price of anarchy under different objective functions of the agents of the network. In both approaches, it is essential to exploit the properties of a temporal network.

For a more concrete example, consider the following problem. We are given a temporal graph $\mathcal{G} = (G, \mathcal{E})$, where $G = (V, E)$ is the underlying static graph and a set of sources $S \subseteq V$. We want to find the minimum number of label changes needed such that the average time needed over all sources to reach every other vertex in the graph is minimized. More formally, our solution is going to be a modified temporal graph $\mathcal{G}$' that achieves the following objective:

$$\min_{\mathcal{G}' \in \tilde{\mathcal{G}}(E(G))} \operatorname*{\texttt{avg}}_{v \in S} \texttt{reachtime}(v, \mathcal{G}').$$

What we expect from you You should bring the curiosity and willingness to delve into aa theoretical research topic in Graph Algorithms and Algorithmic Game Theory.

What you can expect from us We will gently introduce you to the field and accompany you all along this exciting journey. This will be a team effort, and we aim at publishing our results at a renowned international conference.

How to contact us You're welcome to visit us on floor K-2 or send us an e-mail:

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[1] Kitty Meeks: Reducing Reachability in Temporal Graphs: Towards a More Realistic Model of Real-World Spreading Processes (CiE), 2022, pp 186–195
https://link.springer.com/chapter/10.1007/978-3-031-08740-0_16