Algorithm Engineering Group
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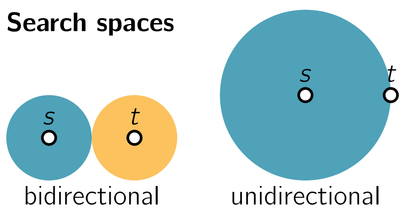
Bidirectional Search in a Realistic Graph Model

Motivation In the analysis of algorithms, a major discrepancy between theory and practice comes from the fact that the analysis usually assumes worst-case instances while real-world instances behave very differently. One example is the bidirectional search, which computes the shortest path between two vertices s and t in a graph by starting a breadth-first search (BFS) from s and from t while stopping when the two search spaces meet. In the worst-case, this does not improve the running time compared to a single BFS starting at s. However, it has been observed that the bidirectional search leads to a significant speed-up on large real-world networks such as social networks or communication networks [1].

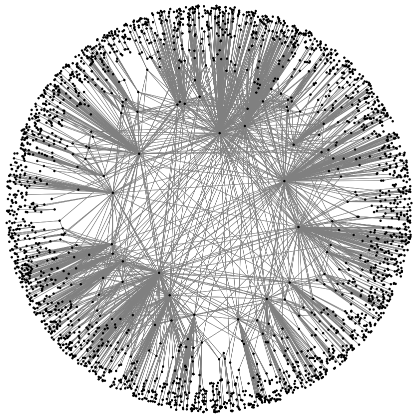


One approach to bridge this gap between theory and practice is to make an average-case analysis by bounding the expected run time under the assumption that the input is randomly drawn from a certain distribution. The practical relevance and explanatory power of such an average-case analysis of course heavily depends on the assumed probability distribution of the input. In case of the bidirectional search, it is actually known that it has a significantly sublinear run time on numerous random graph models [1]. However, the analysis heavily relies on the fact that the edges in the random models under consideration are statistically independent. Unfortunately, this is a very unrealistic assumption: in a social network, two friends of a single person are much more likely to be friends of each other than two random people. Thus, the question why bidirectional search performs so well on large real-world networks remains unanswered.

Purpose of the project We want to theoretically explain why the bidirectional search performs so well on large real-world networks. To this end, we want to bound its expected run time on a more realistic model for such networks, namely on hyperbolic random graphs.

Hyperbolic random graphs When thinking of complex real-world networks, there are two fundamental properties that come to mind. First, they are typically heterogeneous, i.e., they usually have some high-degree and many low-degree vertices (think of few very popular and many "normal" people in a social network). In fact, many real-world networks are *scale-free*, which means that the number of vertices of degree at most x is roughly proportional to $x^{-\beta}$. One also says that such graphs have a *power-law* degree distribution with *power-law exponent* β. Second, vertices with a common neighbor should be more likely to be connected than vertices without common neighbors. Formally, this property can be expressed via the so-called *clustering coefficient*, which, roughly speaking, is the probability that two vertices with a common neighbor are connected.

Hyperbolic random graphs fit the bill as they combine a power-law degree distribution with a high clustering coefficient [2]. They are generated by placing vertices randomly in the hyperbolic plane and connecting two vertices if and only if their hyperbolic distance is small. Note that two vertices that have a common neighbor are geometrically close to this neighbor and thus also close to each other, which increases their probability to be connected and leads to a large clustering coefficient. Moreover, choosing the hyperbolic geometry (instead of the Euclidean) leads to the desired degree distribution.



What we expect from you We expect basic proficiency in the analysis of algorithms, a familiarity with graph-theoretic concepts and interest in randomization and geometry. Prior knowledge about hyperbolic geometry is not required. You should bring the curiosity and willingness to delve into an interesting research topic within Theoretical Computer Science. Our main goal is a rigorous mathematical understanding of algorithms on random graphs and we expect you to contribute theoretical results.

What you can expect from us We will gently introduce you to the field and accompany you all along the interesting journey. This will be a team effort, and we aim at publishing our results at a renowned international conference.

How to contact us You're welcome to visit us on floor A-1 or send us an e-mail:

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[1] Michele Borassi, Emanuele Natale: KADABRA is an ADaptive Algorithm for Betweenness via Random Approximation, ESA 2016 http://dx.doi.org/10.4230/LIPIcs.ESA.2016.20.

[2] Dmitri Krioukov, Fragkiskos Papadopoulos, Maksim Kitsak, Amin Vahdat, and Marián Boguñá: Hyperbolic Geometry of Complex Networks, Phys. Rev. E 82 https://doi.org/10.1103/PhysRevE.82.036106