

Publications of Pascal Lenzner

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Journal articles

- [1] Sauer, P., Cseh, Á., Lenzner, P., Improving ranking quality and fairness in Swiss-system chess tournaments. In: *Journal of Quantitative Analysis in Sports*, 2024.
- [2] Bilò, D., Friedrich, T., Lenzner, P., Melnichenko, A., [Geometric Network Creation Games](#). In: *SIAM Journal on Discrete Mathematics* 38, pp. 277–315, 2024.

Network creation games are a well-known approach for explaining and analyzing the structure, quality, and dynamics of real-world networks that evolved via the interaction of selfish agents without a central authority. In these games selfish agents corresponding to nodes in a network strategically buy incident edges to improve their centrality. However, past research on these games only considered the creation of networks with unit-weight edges. In practice, e.g., when constructing a fiber-optic network, the choice of which nodes to connect and also the induced price for a link crucially depend on the distance between the involved nodes, and such settings can be modeled via edge-weighted graphs. We incorporate arbitrary edge weights by generalizing the well-known model by Fabrikant et al. [Proceedings of PODC '03, ACM, 2003, pp. 347–351] to edge-weighted host graphs and focus on the geometric setting where the weights are induced by the distances in some metric space. In stark contrast to the state of the art for the unit-weight version, where the price of anarchy is conjectured to be constant and where resolving this is a major open problem, we prove a tight nonconstant bound on the price of anarchy for the metric version and a slightly weaker upper bound for the nonmetric case. Moreover, we analyze the existence of equilibria, the computational hardness, and the game dynamics for several natural metrics. The model we propose can be seen as the game-theoretic analogue of the classical network design problem. Thus, low-cost equilibria of our game correspond to decentralized and stable approximations of the optimum network design.

- [3] Friedrich, T., Gawendowicz, H., Lenzner, P., Melnichenko, A., [Social Distancing Network Creation](#). In: *Algorithmica*, 2023.

During a pandemic people have to find a trade-off between meeting others and staying safely at home. While meeting others is pleasant, it also increases the risk of infection. We consider this dilemma by introducing a game-theoretic network creation model in which selfish agents can form bilateral connections. They benefit from network neighbors, but at the same time, they want to maximize their distance to all other agents. This models the inherent conflict that social distancing rules impose on the behavior of selfish agents in a social network. Besides addressing this familiar issue, our model can be seen as the inverse to the well-studied Network Creation Game by Fabrikant et al. (in: PODC 2003, pp 347–351, 2003. <https://doi.org/10.1145/872035.872088>), where agents aim at being as central as possible in the created network. We look at two variants of network creation governed by social distancing. Firstly, a variant without connection restrictions, where we characterize optimal and equilibrium networks, and derive asymptotically tight bounds on the Price of Anarchy and Price of Stability. The second variant allows connection restrictions. As our main result, we prove that Swap-Maximal Routing-Cost Spanning Trees, an efficiently computable weaker variant of Maximum Routing-Cost Spanning Trees, actually resemble equilibria for a significant range of the parameter space. Moreover, we give almost tight bounds on the Price of Anarchy and Price of Stability. These results imply that under social distancing the agents' selfishness has a strong impact on the quality of the equilibria.

- [4] Bilò, D., Bilò, V., Lenzner, P., Molitor, L., [Topological Influence and Locality in Swap Schelling Games](#). In: *Autonomous Agents and Multi-Agent Systems (AGNT)* 36, pp. 47, 2022.

Residential segregation is a wide-spread phenomenon that can be observed in almost every major city. In these urban areas residents with different racial or socioeconomic background tend to form homogeneous clusters. Schelling's famous agent-based model for residential segregation explains how such clusters can form even if all agents are tolerant, i.e., if they agree to live in mixed neighborhoods. For segregation to occur, all it needs is a slight bias towards agents preferring similar neighbors. Very recently, Schelling's model has been investigated from a game-theoretic point of view with selfish agents that strategically select their residential location. In these games, agents can improve on their current location by performing a location swap with another agent who is willing to swap. We significantly deepen these investigations by studying the influence of the underlying topology modeling the residential area on the existence of equilibria, the Price of Anarchy and on the dynamic properties of the resulting strategic multi-agent system. Moreover, as a new conceptual contribution, we also consider the influence of locality, i.e., if the location swaps are restricted to swaps of neighboring agents. We give improved almost tight bounds on the Price of Anarchy for arbitrary underlying graphs and we present (almost) tight bounds for regular graphs, paths and cycles. Moreover, we give almost tight bounds for grids, which are commonly used in empirical studies. For grids we also show that locality has a severe impact on the game dynamics.

- [5] Bilò, D., Lenzner, P., [On the Tree Conjecture for the Network Creation Game](#). In: *Theory of Computing Systems* 64, pp. 422–443, 2020.

Selfish Network Creation focuses on modeling real world networks from a game-theoretic point of view. One of the classic models by Fabrikant et al. (2003) is the network creation game, where agents correspond to nodes in a network which buy incident edges for the price of α per edge to minimize their total distance to all other nodes. The model is well-studied but still has intriguing open problems. The most famous conjectures state that the price of anarchy is constant for all α and that for $\alpha \geq n$ all equilibrium networks are trees. We introduce a novel technique for analyzing stable networks for high edge-price α and employ it to improve on the best known bound for the latter conjecture. In particular we show that for $\alpha > 4n - 13$ all equilibrium networks must be trees, which implies a constant price of anarchy for this range of α . Moreover, we also improve the constant upper bound on the price of anarchy for equilibrium trees.

- [6] Albers, S., Lenzner, P., [On Approximate Nash Equilibria in Network Design](#). In: *Internet Mathematics* 9, pp. 384–405, 2013.

We study a basic network design game where n self-interested agents, each having individual connectivity requirements, wish to build a network by purchasing links from a given set of edges. A fundamental cost sharing mechanism is Shapley cost sharing that splits the cost of an edge in a fair manner among the agents using the edge. In this paper we investigate if an optimal minimum-cost network represents an attractive, relatively stable state that agents might want to purchase. We resort to the concept of α -approximate Nash equilibria. We prove that for single source games in undirected graphs, any optimal network represents an $H(n)$ -approximate Nash equilibrium, where $H(n)$ is the n -th Harmonic number. We show that this bound is tight. We extend the results to cooperative games, where agents may form coalitions, and to weighted games. In both cases we give tight or nearly tight lower and upper bounds on the stability of optimal solutions. Finally we show that in general source-sink games and in directed graphs, minimum-cost networks do not represent good states.

Conference papers

- [7] Friedrich, T., Gawendowicz, H., Lenzner, P., Zahn, A., [The Impact of Cooperation in Bilateral Network Creation](#). In: *ACM Symposium on Principles of Distributed Computing (PODC)*, 2023.

Many real-world networks, like the Internet or social networks, are not the result of central design but instead the outcome of the interaction of local agents that selfishly optimize their individual utility. The well-known Network Creation Game by Fabrikant, Luthra, Maneva, Papadimitriou, and Shenker [PODC 2003] models this. There, agents corresponding to network nodes buy incident edges towards other agents for a price of $\alpha > 0$ and simultaneously try to minimize their buying cost and their total hop distance. Since in many real-world networks, e.g., social networks, consent from both sides is required to establish and maintain a connection, Corbo and Parkes [PODC 2005] proposed a bilateral version of the Network Creation Game, in which mutual consent and payment are required in order to create edges. It is known that this cooperative version has a significantly higher Price of Anarchy compared to the unilateral version. On the first glance this is counter-intuitive, since cooperation should help to avoid socially bad states. However, in the bilateral version only a very restrictive form of cooperation is considered. We investigate this trade-off between the amount of cooperation and the Price of Anarchy by analyzing the bilateral version with respect to various degrees of cooperation among the agents. With this, we provide insights into what kind of cooperation is needed to ensure that socially good networks are created. As a first step in this direction, we focus on tree networks and present a collection of asymptotically tight bounds on the Price of Anarchy that precisely map the impact of cooperation. Most strikingly, we find that weak forms of cooperation already yield a significantly improved Price of Anarchy. In particular, the cooperation of coalitions of size 3 is enough to achieve constant bounds. Moreover, for general networks we show that enhanced cooperation yields close to optimal networks for a wide range of edge prices. Along the way, we disprove an old conjecture by Corbo and Parkes [PODC 2005].

- [8] Cseh, Á., Führlich, P., Lenzner, P., [The Swiss Gambit](#). In: *Autonomous Agents and Multi-Agent Systems (AAMAS)*, 2023.

In each round of a Swiss-system tournament, players of similar score are paired against each other. An intentional early loss therefore might lead to weaker opponents in later rounds and thus to a better final tournament result a phenomenon known as the Swiss Gambit. To the best of our knowledge it is an open question whether this strategy can actually work. This paper provides answers based on an empirical agent-based analysis for the most prominent application area of the Swiss-system format, namely chess tournaments. We simulate realistic tournaments by employing the official FIDE pairing system for computing the player pairings in each round. We show that even though gambits are widely possible in Swiss-system chess tournaments, profiting from them requires a high degree of predictability of match results. Moreover, even if a Swiss Gambit succeeds, the obtained improvement in the final ranking is limited. Our experiments prove that counting on a Swiss Gambit is indeed a lot more of a risky gambit than a reliable strategy to improve the final rank.

- [9] Bertschinger, N., Hoefler, M., Krogmann, S., Lenzner, P., Schuldensucker, S., Wilhelmi, L., [Equilibria and Convergence in Fire Sale Games](#). In: *Autonomous Agents and Multiagent Systems (AAMAS)*, pp. 215–223, 2023.

The complex interactions between algorithmic trading agents can have a severe influence on the functioning of our economy, as witnessed by recent banking crises and trading anomalies. A common phenomenon in these situations are fire sales, a contagious process of asset sales that trigger further sales. We study the existence and structure of equilibria in a game-theoretic model of fire sales. We prove that for a wide parameter range (e.g., convex price impact functions), equilibria exist and form a complete lattice. This is contrasted with a non-existence result for concave price impact functions. Moreover, we study the convergence of best-response dynamics towards equilibria when they exist. In general, best-response dynamics may cycle. However, in many settings they are guaranteed to converge to the socially optimal equilibrium when starting from a natural initial state. Moreover, we discuss a simplified variant of the dynamics that is less informationally demanding and converges to the same equilibria. We compare the dynamics in terms of convergence speed.

- [10] Krogmann, S., Lenzner, P., Skopalik, A., [Strategic Facility Location with Clients that Minimize Total Waiting Time](#). In: *Conference on Artificial Intelligence (AAAI)*, 2023.

We study a non-cooperative two-sided facility location game in which facilities and clients behave strategically. This is in contrast to many other facility location games in which clients simply visit their closest facility. Facility agents select a location on a graph to open a facility to attract as much purchasing power as possible, while client agents choose which facilities to patronize by strategically distributing their purchasing power in order to minimize their total waiting time. Here, the waiting time of a facility depends on its received total purchasing power. We show that our client stage is an atomic splittable congestion game, which implies existence, uniqueness and efficient computation of a client equilibrium. Therefore, facility agents can efficiently predict client behavior and make strategic decisions accordingly. Despite that, we prove that subgame perfect equilibria do not exist in all instances of this game and that their existence is NP-hard to decide. On the positive side, we provide a simple and efficient algorithm to compute 3-approximate subgame perfect equilibria.

- [11] Bilò, D., Bilò, V., Döring, M., Lenzner, P., Molitor, L., Schmidt, J., [Schelling Games with Continuous Types](#). In: *International Joint Conference on Artificial Intelligence (IJCAI)*, pp. 2520–2527, 2023.

In most major cities and urban areas, residents form homogeneous neighborhoods along ethnic or socioeconomic lines. This phenomenon is widely known as residential segregation and has been studied extensively. Fifty years ago, Schelling proposed a landmark model that explains residential segregation in an elegant agent-based way. A recent stream of papers analyzed Schelling’s model using game-theoretic approaches. However, all these works considered models with a given number of discrete types modeling different ethnic groups. We focus on segregation caused by non-categorical attributes, such as household income or position in a political left-right spectrum. For this, we consider agent types that can be represented as real numbers. This opens up a great variety of reasonable models and, as a proof of concept, we focus on several natural candidates. In particular, we consider agents that evaluate their location by the average type-difference or the maximum type-difference to their neighbors, or by having a certain tolerance range for type-values of neighboring agents. We study the existence and computation of equilibria and provide bounds on the Price of Anarchy and Stability. Also, we present simulation results that compare our models and shed light on the obtained equilibria for our variants.

- [12] Bilò, D., Cohen, S., Friedrich, T., Gawendowicz, H., Klodt, N., Lenzner, P., Skretas, G., [Temporal Network Creation Games](#). In: *International Joint Conference on Artificial Intelligence (IJCAI)*, pp. 2511–2519, 2023.

Most networks are not static objects, but instead they change over time. This observation has sparked rigorous research on temporal graphs within the last years. In temporal graphs, we have a fixed set of nodes and the connections between them are only available at certain time steps. This gives rise to a plethora of algorithmic problems on such graphs, most prominently the problem of finding temporal spanners, i.e., the computation of subgraphs that guarantee all pairs reachability via temporal paths. To the best of our knowledge, only centralized approaches for the solution of this problem are known. However, many real-world networks are not shaped by a central designer but instead they emerge and evolve by the interaction of many strategic agents. This observation is the driving force of the recent intensive research on game-theoretic network formation models. In this work we bring together these two recent research directions: temporal graphs and game-theoretic network formation. As a first step into this new realm, we focus on a simplified setting where a complete temporal host graph is given and the agents, corresponding to its nodes, selfishly create incident edges to ensure that they can reach all other nodes via temporal paths in the created network. This yields temporal spanners as equilibria of our game. We prove results on the convergence to and the existence of equilibrium networks, on the complexity of finding best agent strategies, and on the quality of the equilibria. By taking these first important steps, we uncover challenging open problems that call for an in-depth exploration of the creation of temporal graphs by strategic agents.

- [13] Gadea Harder, J., Krogmann, S., Lenzner, P., Skopalik, A., [Strategic Resource Selection with Homophilic Agents](#). In: *International Joint Conference on Artificial Intelligence (IJCAI)*, 2023.

The strategic selection of resources by selfish agents is a classic research direction, with Resource Selection Games and Congestion Games as prominent examples. In these games, agents select available resources and their utility then depends on the number of agents using the same resources. This implies that there is no distinction between the agents, i.e., they are anonymous. We depart from this very general setting by proposing Resource Selection Games with heterogeneous agents that strive for joint resource usage with similar agents. So, instead of the number of other users of a given resource, our model considers agents with different types and the decisive feature is the fraction of same-type agents among the users. More precisely, similarly to Schelling Games, there is a tolerance threshold $\tau \in [0, 1]$ which specifies the agents’ desired minimum fraction of same-type agents on a resource. Agents strive to select resources where at least a τ -fraction of those resources’ users have the same type as themselves. For $\tau = 1$, our model generalizes Hedonic Diversity Games with a peak at 1. For our general model, we consider the existence and quality of equilibria and the complexity of maximizing social welfare. Additionally, we consider a bounded rationality model, where agents can only estimate the utility of a resource, since they only know the fraction of same-type agents on a given resource, but not the exact numbers. Thus, they cannot know the impact a strategy change would have on a target resource. Interestingly, we show that this type of bounded rationality yields favorable game-theoretic properties and specific equilibria closely approximate equilibria of the full knowledge setting.

- [14] Khomutovskiy, I., Dunker, R., Dierking, J., Egbert, J., Helms, C., Schöllkopf, F., Casel, K., Fischbeck, P., Friedrich, T., Isaac, D., Krogmann, S., Lenzner, P., [Applying Skeletons to Speed Up the Arc-Flags Routing Algorithm](#). In: *SIAM Symposium on Algorithm Engineering and Experiments (ALENEX)*, pp. 110–122, 2023.

The Single-Source Shortest Path problem is classically solved by applying Dijkstra’s algorithm. However, the plain version of this algorithm is far too slow for real-world applications such as routing in large road networks. To amend this, many speed-up techniques have been developed that build on the idea of computing auxiliary data in a preprocessing phase, that is used to speed up the queries. One well-known example is the Arc-Flags algorithm that is based on the idea of precomputing edge flags to make the search more goal-directed. To explain the strong practical performance of such speed-up techniques, several graph parameters have been introduced. The skeleton dimension is one such parameter that has already been used to derive runtime bounds for some speed-up techniques. Moreover, it was experimentally shown to be low in real-world road networks. We introduce a method to incorporate skeletons, the underlying structure behind the skeleton dimension, to improve routing speed-up techniques even further. As a proof of concept, we develop new algorithms called SKARF and SKARF+ that combine skeletons with Arc-Flags, and demonstrate via extensive experiments on large real-world road networks that SKARF+ yields a significant reduction of the search space and the query time of about 30% to 40% over Arc-Flags. We also prove theoretical bounds on the query time of SKARF, which is the first time an Arc-Flags variant has been analyzed in terms of skeleton dimension.

- [15] Friedrich, T., Lenzner, P., Molitor, L., Seifert, L., [Single-Peaked Jump Schelling Games](#). In: 2023.

Schelling games model the wide-spread phenomenon of residential segregation in metropolitan areas from a game-theoretic point of view. In these games agents of different types each strategically select a node on a given graph that models the residential area to maximize their individual utility. The latter solely depends on the types of the agents on neighboring nodes and it has been a standard assumption to consider utility functions that are monotone in the number of same-type neighbors. This simplifying assumption has recently been challenged since sociological poll results suggest that real-world agents actually favor diverse neighborhoods. We contribute to the recent endeavor of investigating residential segregation models with realistic agent behavior by studying Jump Schelling Games with agents having a single-peaked utility function. In such games, there are empty nodes in the graph and agents can strategically jump to such nodes to improve their utility. We investigate the existence of equilibria and show that they exist under specific conditions. Contrasting this, we prove that even on simple topologies like paths or rings such stable states are not guaranteed to exist. Regarding the game dynamics, we show that improving response cycles exist independently of the position of the peak in the utility function. Moreover, we show high almost tight bounds on the Price of Anarchy and the Price of Stability with respect to the recently proposed degree of integration, which counts the number of agents with a diverse neighborhood and which serves as a proxy for measuring the segregation strength. Last but not least, we show that computing a beneficial state with high integration is NP-complete and, as a novel conceptual contribution, we also show that it is NP-hard to decide if an equilibrium state can be found via improving response dynamics starting from a given initial state.

- [16] Führlich, P., Cseh, Á., Lenzner, P., [Improving Ranking Quality and Fairness in Swiss-System Chess Tournaments](#). In: *ACM Conference on Economics and Computation (EC)*, pp. 1101–1102, 2022.

The International Chess Federation (FIDE) imposes a voluminous and complex set of player pairing criteria in Swiss-system chess tournaments and endorses computer programs that are able to calculate the prescribed pairings. The purpose of these formalities is to ensure that players are paired fairly during the tournament and that the final ranking corresponds to the players' true strength order. We contest the official FIDE player pairing routine by presenting alternative pairing rules. These can be enforced by computing maximum weight matchings in a carefully designed graph. We demonstrate by extensive experiments that a tournament format using our mechanism (1) yields fairer pairings in the rounds of the tournament and (2) produces a final ranking that reflects the players' true strengths better than the state-of-the-art FIDE pairing system.

- [17] Berenbrink, P., Hofer, M., Kaaser, D., Lenzner, P., Rau, M., Schmand, D., [Asynchronous Opinion Dynamics in Social Networks](#). In: *Autonomous Agents and Multi-Agent Systems (AAMAS)*, pp. 109–117, 2022.

Opinion spreading in a society decides the fate of elections, the success of products, and the impact of political or social movements. The model by Hegselmann and Krause is a well-known theoretical model to study such opinion formation processes in social networks. In contrast to many other theoretical models, it does not converge towards a situation where all agents agree on the same opinion. Instead, it assumes that people find an opinion reasonable if and only if it is close to their own. The system converges towards a stable situation where agents sharing the same opinion form a cluster, and agents in different clusters do not influence each other. We focus on the social variant of the Hegselmann-Krause model where agents are connected by a social network and their opinions evolve in an iterative process. When activated, an agent adopts the average of the opinions of its neighbors having a similar opinion. By this, the set of influencing neighbors of an agent may change over time. To the best of our knowledge, social Hegselmann-Krause systems with asynchronous opinion updates have only been studied with the complete graph as social network. We show that such opinion dynamics with random agent activation are guaranteed to converge for any social network. We provide an upper bound of $\mathcal{O}(n|E|^2(\varepsilon/\delta)^2)$ on the expected number of opinion updates until convergence, where $|E|$ is the number of edges of the social network. For the complete social network we show a bound of $\mathcal{O}(n^3(n^2 + (\varepsilon/\delta)^2))$ that represents a major improvement over the previously best upper bound of $\mathcal{O}(n^9(\varepsilon/\delta)^2)$. Our bounds are complemented by simulations that indicate asymptotically matching lower bounds.

- [18] Friedrich, T., Gawendowicz, H., Lenzner, P., Melnichenko, A., [Social Distancing Network Creation](#). In: *International Colloquium on Automata, Languages and Programming (ICALP)*, pp. 62:1–62:21, 2022.

During a pandemic people have to find a trade-off between meeting others and staying safely at home. While meeting others is pleasant, it also increases the risk of infection. We consider this dilemma by introducing a game-theoretic network creation model in which selfish agents can form bilateral connections. They benefit from network neighbors, but at the same time, they want to maximize their distance to all other agents. This models the inherent conflict that social distancing rules impose on the behavior of selfish agents in a social network. Besides addressing this familiar issue, our model can be seen as the inverse to the well-studied Network Creation Game by Fabrikant et al. [PODC 2003] where agents aim at being as central as possible in the created network. Thus, our work is in-line with studies that compare minimization problems with their maximization versions. We look at two variants of network creation governed by social distancing. In the first variant, there are no restrictions on the connections being formed. We characterize optimal and equilibrium networks, and we derive asymptotically tight bounds on the Price of Anarchy and Price of Stability. The second variant is the model's generalization that allows restrictions on the connections that can be formed. As our main result, we prove that Swap-Maximal Routing-Cost Spanning Trees, an efficiently computable weaker variant of Maximum Routing-Cost Spanning Trees, actually resemble equilibria for a significant range of the parameter space. Moreover, we give almost tight bounds on the Price of Anarchy and Price of Stability. These results imply that, compared the well-studied inverse models, under social distancing the agents' selfish behavior has a significantly stronger impact on the quality of the equilibria, i.e., allowing socially much worse stable states.

- [19] Bullinger, M., Lenzner, P., Melnichenko, A., [Network Creation with Homophilic Agents](#). In: *International Joint Conference on Artificial Intelligence (IJCAI)*, pp. 151–157, 2022.

Network Creation Games are an important framework for understanding the formation of real-world networks such as social networks. These games usually assume a set of indistinguishable agents strategically buying edges at a uniform price leading to a network among them. However, in real life, agents are heterogeneous and their relationships often display a bias towards similar agents, say of the same ethnic group. This homophilic behavior on the agent level can then lead to the emergent global phenomenon of social segregation. We initiate the study of Network Creation Games with multiple types of homophilic agents and non-uniform edge cost. Specifically, we introduce and compare two models, focusing on the perception of same-type and different-type neighboring agents, respectively. Despite their different initial conditions, both our theoretical and experimental analysis show that the resulting equilibrium networks are almost identical in the two models, indicating a robust structure of social networks under homophily. Moreover, we investigate the segregation strength of the formed networks and thereby offer new insights on understanding segregation.

- [20] Bilò, D., Bilò, V., Lenzner, P., Molitor, L., [Tolerance is Necessary for Stability: Single-Peaked Swap Schelling Games](#). In: *International Joint Conference on Artificial Intelligence (IJCAI)*, pp. 81–87, 2022.

Residential segregation in metropolitan areas is a phenomenon that can be observed all over the world. Recently, this was investigated via game-theoretic models. There, selfish agents of two types are equipped with a monotone utility function that ensures higher utility if an agent has more same-type neighbors. The agents strategically choose their location on a given graph that serves as residential area to maximize their utility. However, sociological polls suggest that real-world agents are actually favoring mixed-type neighborhoods, and hence should be modeled via non-monotone utility functions. To address this, we study Swap Schelling Games with single-peaked utility functions. Our main finding is that tolerance, i.e., agents favoring fifty-fifty neighborhoods or being in the minority, is necessary for equilibrium existence on almost regular or bipartite graphs. Regarding the quality of equilibria, we derive (almost) tight bounds on the Price of Anarchy and the Price of Stability. In particular, we show that the latter is constant on bipartite and almost regular graphs.

- [21] Bilò, D., Friedrich, T., Lenzner, P., Lowski, S., Melnichenko, A., [Selfish Creation of Social Networks](#). In: *Conference on Artificial Intelligence (AAAI)*, pp. 5185–5193, 2021.

Understanding real-world networks has been a core research endeavor throughout the last two decades. Network Creation Games are a promising approach for this from a game-theoretic perspective. In these games, selfish agents corresponding to nodes in a network strategically decide which links to form to optimize their centrality. Many versions have been introduced and analyzed, but none of them fits to modeling the evolution of social networks. In real-world social networks, connections are often established by

recommendations from common acquaintances or by a chain of such recommendations. Thus establishing and maintaining a contact with a friend of a friend is easier than connecting to complete strangers. This explains the high clustering, i.e., the abundance of triangles, in real-world social networks. We propose and analyze a network creation model inspired by real-world social networks. Edges are formed in our model via bilateral consent of both endpoints and the cost for establishing and maintaining an edge is proportional to the distance of the endpoints before establishing the connection. We provide results for generic cost functions, which essentially only must be convex functions in the distance of the endpoints without the respective edge. For this broad class of cost functions, we provide many structural properties of equilibrium networks and prove (almost) tight bounds on the diameter, the Price of Anarchy and the Price of Stability. Moreover, as a proof-of-concept we show via experiments that the created equilibrium networks of our model indeed closely mimics real-world social networks. We observe degree distributions that seem to follow a power-law, high clustering, and low diameters. This can be seen as a promising first step towards game-theoretic network creation models that predict networks featuring all core real-world properties.

- [22] Krogmann, S., Lenzner, P., Molitor, L., Skopalik, A., [Two-Stage Facility Location Games with Strategic Clients and Facilities](#). In: *International Joint Conference on Artificial Intelligence (IJCAI)*, pp. 292–298, 2021.

We consider non-cooperative facility location games where both facilities and clients act strategically and heavily influence each other. This contrasts established game-theoretic facility location models with non-strategic clients that simply select the closest opened facility. In our model, every facility location has a set of attracted clients and each client has a set of shopping locations and a weight that corresponds to her spending capacity. Facility agents selfishly select a location for opening their facility to maximize the attracted total spending capacity, whereas clients strategically decide how to distribute their spending capacity among the opened facilities in their shopping range. We focus on a natural client behavior similar to classical load balancing: our selfish clients aim for a distribution that minimizes their maximum waiting times for getting serviced, where a facility’s waiting time corresponds to its total attracted client weight. We show that subgame perfect equilibria exist and give almost tight constant bounds on the Price of Anarchy and the Price of Stability, which even hold for a broader class of games with arbitrary client behavior. Since facilities and clients influence each other, it is crucial for the facilities to anticipate the selfish clients’ behavior when selecting their location. For this, we provide an efficient algorithm that also implies an efficient check for equilibrium. Finally, we show that computing a socially optimal facility placement is NP-hard and that this result holds for all feasible client weight distributions.

- [23] Friedemann, W., Friedrich, T., Gawendowicz, H., Lenzner, P., Melnichenko, A., Peters, J., Stephan, D., Vaichenker, M., [Efficiency and Stability in Euclidean Network Design](#). In: *Symposium on Parallelism in Algorithms and Architectures (SPAA)*, pp. 232–242, 2021.

Network Design problems typically ask for a minimum cost sub-network from a given host network. This classical point-of-view assumes a central authority enforcing the optimum solution. But how should networks be designed to cope with selfish agents that own parts of the network? In this setting, minimum cost networks may be very unstable in that agents will deviate from a proposed solution if this decreases their individual cost. Hence, designed networks should be both efficient in terms of total cost and stable in terms of the agents’ willingness to accept the network. We study this novel type of Network Design problem by investigating the creation of (β, γ) -networks, that are in β -approximate Nash equilibrium and have a total cost of at most γ times the optimal cost, for the recently proposed Euclidean Generalized Network Creation Game by Bilò et al. SPAA2019. There, n agents corresponding to points in Euclidean space create costly edges among themselves to optimize their centrality in the created network. Our main result is a simple $\mathcal{O}(n^2)$ -time algorithm that computes a (β, β) -network with low β for any given set of points. Moreover, on integer grid point sets or random point sets our algorithm achieves a low constant β . Besides these results for the Euclidean model, we discuss a generalization of our algorithm to instances with arbitrary, even non-metric, edge lengths. Moreover, in contrast to these algorithmic results, we show that no such positive results are possible when focusing on either optimal networks, i.e., $(\beta, 1)$ -networks, or perfectly stable networks, i.e., $(1, \gamma)$ -networks, as in both cases NP-hard problems arise, there exist instances with very unstable optimal networks, and there are instances for perfectly stable networks with high total cost. Along the way, we significantly improve several results from Bilò et al. and we asymptotically resolve their conjecture about the Price of Anarchy by providing a tight bound.

- [24] Bläsius, T., Böther, M., Fischbeck, P., Friedrich, T., Gries, A., Hüffner, F., Kießig, O., Lenzner, P., Molitor, L., Schiller, L., Wells, A., Wietheger, S., [A Strategic Routing Framework and Algorithms for Computing Alternative Paths](#). In: *Algorithmic Approaches for Transportation Modelling, Optimization, and Systems (ATMOS)*, pp. 10:1–10:14, 2020.

Traditional navigation services find the fastest route for a single driver. Though always using the fastest route seems desirable for every individual, selfish behavior can have undesirable effects such as higher energy consumption and avoidable congestion, even leading to higher overall and individual travel times. In contrast, strategic routing aims at optimizing the traffic for all agents regarding a global optimization goal. We introduce a framework to formalize real-world strategic routing scenarios as algorithmic problems and study one of them, which we call Single Alternative Path (SAP), in detail. There, we are given an original route between a single origin–destination pair. The goal is to suggest an alternative route to all agents that optimizes the overall travel time under the assumption that the agents distribute among both routes according to a psychological model, for which we introduce the concept of Pareto-conformity. We show that the SAP problem is NP-complete, even for such models. Nonetheless, assuming Pareto-conformity, we give multiple algorithms for different variants of SAP, using multi-criteria shortest path algorithms as subroutines. Moreover, we prove that several natural models are in fact Pareto-conform. The implementation and evaluation of our algorithms serve as a proof of concept, showing that SAP can be solved in reasonable time even though the algorithms have exponential running time in the worst case.

- [25] Bilò, D., Friedrich, T., Lenzner, P., Melnichenko, A., Molitor, L., [Fair Tree Connection Games with Topology-Dependent Edge Cost](#). In: *Foundations of Software Technology and Theoretical Computer Science (FSTTCS)*, pp. 15:1–15:15, 2020.

How do rational agents self-organize when trying to connect to a common target? We study this question with a simple tree formation game which is related to the well-known fair single-source connection game by Anshelevich et al. (FOCS’04) and selfish spanning tree games by Gourvès and Monnot (WINE’08). In our game agents correspond to nodes in a network that activate a single outgoing edge to connect to the common target node (possibly via other nodes). Agents pay for their path to the common target, and edge costs are shared fairly among all agents using an edge. The main novelty of our model is dynamic edge costs that depend on the in-degree of the respective endpoint. This reflects that connecting to popular nodes that have increased internal coordination costs is more expensive since they can charge higher prices for their routing service. In contrast to related models, we show that equilibria are not guaranteed to exist, but we prove the existence for infinitely many numbers of agents. Moreover, we analyze the structure of equilibrium trees and employ these insights to prove a constant upper bound on the Price of Anarchy as well as non-trivial lower bounds on both the Price of Anarchy and the Price of Stability. We also show that in comparison with the social optimum tree the overall cost of an equilibrium

tree is more fairly shared among the agents. Thus, we prove that self-organization of rational agents yields on average only slightly higher cost per agent compared to the centralized optimum, and at the same time, it induces a more fair cost distribution. Moreover, equilibrium trees achieve a beneficial trade-off between a low height and low maximum degree, and hence these trees might be of independent interest from a combinatorics point-of-view. We conclude with a discussion of promising extensions of our model.

- [26] Echzell, H., Friedrich, T., Lenzner, P., Melnichenko, A., [Flow-Based Network Creation Games](#). In: *International Joint Conference on Artificial Intelligence (IJCAI)*, pp. 139–145, 2020.

Network Creation Games (NCGs) model the creation of decentralized communication networks like the Internet. In such games strategic agents corresponding to network nodes selfishly decide with whom to connect to optimize some objective function. Past research intensively analyzed models where the agents strive for a central position in the network. This models agents optimizing the network for low-latency applications like VoIP. However, with today’s abundance of streaming services it is important to ensure that the created network can satisfy the increased bandwidth demand. To the best of our knowledge, this natural problem of the decentralized strategic creation of networks with sufficient bandwidth has not yet been studied. We introduce Flow-Based NCGs where the selfish agents focus on bandwidth instead of latency. In essence, budget-constrained agents create network links to maximize their minimum or average network flow value to all other network nodes. Equivalently, this can also be understood as agents who create links to increase their connectivity and thus also the robustness of the network. For this novel type of NCG we prove that pure Nash equilibria exist, we give a simple algorithm for computing optimal networks, we show that the Price of Stability is 1 and we prove an (almost) tight bound of 2 on the Price of Anarchy. Last but not least, we show that our models do not admit a potential function.

- [27] Bilò, D., Bilò, V., Lenzner, P., Molitor, L., [Topological Influence and Locality in Swap Schelling Games](#). In: *International Symposium on Mathematical Foundations of Computer Science (MFCS)*, pp. 15:1–15:15, 2020.

Residential segregation is a wide-spread phenomenon that can be observed in almost every major city. In these urban areas residents with different racial or socioeconomic background tend to form homogeneous clusters. Schelling’s famous agent-based model for residential segregation explains how such clusters can form even if all agents are tolerant, i.e., if they agree to live in mixed neighborhoods. For segregation to occur, all it needs is a slight bias towards agents preferring similar neighbors. Very recently, Schelling’s model has been investigated from a game-theoretic point of view with selfish agents that strategically select their residential location. In these games, agents can improve on their current location by performing a location swap with another agent who is willing to swap. We significantly deepen these investigations by studying the influence of the underlying topology modeling the residential area on the existence of equilibria, the Price of Anarchy and on the dynamic properties of the resulting strategic multi-agent system. Moreover, as a new conceptual contribution, we also consider the influence of locality, i.e., if the location swaps are restricted to swaps of neighboring agents. We give improved almost tight bounds on the Price of Anarchy for arbitrary underlying graphs and we present (almost) tight bounds for regular graphs, paths and cycles. Moreover, we give almost tight bounds for grids, which are commonly used in empirical studies. For grids we also show that locality has a severe impact on the game dynamics.

- [28] Feldotto, M., Lenzner, P., Molitor, L., Skopalik, A., [From Hotelling to Load Balancing: Approximation and the Principle of Minimum Differentiation](#). In: *Autonomous Agents and Multiagent Systems (AAMAS)*, pp. 1949–1951, 2019.

Competing firms tend to select similar locations for their stores. This phenomenon, called the principle of minimum differentiation, was captured by Hotelling with a landmark model of spatial competition but is still the object of an ongoing scientific debate. Although consistently observed in practice, many more realistic variants of Hotelling’s model fail to support minimum differentiation or do not have pure equilibria at all. In particular, it was recently proven for a generalized model which incorporates negative network externalities and which contains Hotelling’s model and classical selfish load balancing as special cases, that the unique equilibria do not adhere to minimum differentiation. Furthermore, it was shown that for a significant parameter range pure equilibria do not exist. We derive a sharp contrast to these previous results by investigating Hotelling’s model with negative network externalities from an entirely new angle: approximate pure subgame perfect equilibria. This approach allows us to prove analytically and via agent-based simulations that approximate equilibria having good approximation guarantees and that adhere to minimum differentiation exist for the full parameter range of the model. Moreover, we show that the obtained approximate equilibria have high social welfare.

- [29] Bilò, D., Friedrich, T., Lenzner, P., Melnichenko, A., [Geometric Network Creation Games](#). In: *Symposium on Parallelism in Algorithms and Architectures (SPAA)*, pp. 323–332, 2019.

Network Creation Games are a well-known approach for explaining and analyzing the structure, quality and dynamics of real-world networks like the Internet and other infrastructure networks which evolved via the interaction of selfish agents without a central authority. In these games selfish agents which correspond to nodes in a network strategically buy incident edges to improve their centrality. However, past research on these games has only considered the creation of networks with unit-weight edges. In practice, e.g. when constructing a fiber-optic network, the choice of which nodes to connect and also the induced price for a link crucially depends on the distance between the involved nodes and such settings can be modeled via edge-weighted graphs. We incorporate arbitrary edge weights by generalizing the well-known model by Fabrikant et al. [PODC’03] to edge-weighted host graphs and focus on the geometric setting where the weights are induced by the distances in some metric space. In stark contrast to the state-of-the-art for the unit-weight version, where the Price of Anarchy is conjectured to be constant and where resolving this is a major open problem, we prove a tight non-constant bound on the Price of Anarchy for the metric version and a slightly weaker upper bound for the non-metric case. Moreover, we analyze the existence of equilibria, the computational hardness and the game dynamics for several natural metrics. The model we propose can be seen as the game-theoretic analogue of a variant of the classical Network Design Problem. Thus, low-cost equilibria of our game correspond to decentralized and stable approximations of the optimum network design.

- [30] Echzell, H., Friedrich, T., Lenzner, P., Molitor, L., Pappik, M., Schöne, F., Sommer, F., Stangl, D., [Convergence and Hardness of Strategic Schelling Segregation](#). In: *Web and Internet Economics (WINE)*, pp. 156–170, 2019.

The phenomenon of residential segregation was captured by Schelling’s famous segregation model where two types of agents are placed on a grid and an agent is content with her location if the fraction of her neighbors which have the same type as her is at least τ , for some $0 < \tau < 1$. Discontent agents simply swap their location with a randomly chosen other discontent agent or jump to a random empty cell. We analyze a generalized game-theoretic model of Schelling segregation which allows more than two agent types and more general underlying graphs modeling the residential area. For this we show that both aspects heavily influence the dynamic properties and the tractability of finding an optimal placement. We map the boundary of when improving response dynamics (IRD), i.e., the natural approach for finding equilibrium states, are guaranteed to converge. For this we prove several sharp threshold results where guaranteed IRD convergence suddenly turns into the strongest possible non-convergence result: a violation of weak acyclicity. In particular, we show such threshold results also for Schelling’s original model, which is in contrast to the standard assumption in

many empirical papers. Furthermore, we show that in case of convergence, IRD find an equilibrium in $O(m)$ steps, where m is the number of edges in the underlying graph and show that this bound is met in empirical simulations starting from random initial agent placements.

- [31] Chauhan, A., Lenzner, P., Molitor, L., [Schelling Segregation with Strategic Agents](#). In: *Symposium on Algorithmic Game Theory (SAGT)*, 2018.

Schelling's segregation model is a landmark model in sociology. It shows the counter-intuitive phenomenon that residential segregation between individuals of different groups can emerge even when all involved individuals are tolerant. Although the model is widely studied, no pure game-theoretic version where rational agents strategically choose their location exists. We close this gap by introducing and analyzing generalized game-theoretic models of Schelling segregation, where the agents can also have individual location preferences. For our models we investigate the convergence behavior and the efficiency of their equilibria. In particular, we prove guaranteed convergence to an equilibrium in the version which is closest to Schelling's original model. Moreover, we provide tight bounds on the Price of Anarchy.

- [32] Bilò, D., Lenzner, P., [On the Tree Conjecture for the Network Creation Game](#). In: *Symposium on the Theoretical Aspects of Computer Science (STACS)*, pp. 14:1–14:15, 2018.

Selfish Network Creation focuses on modeling real world networks from a game-theoretic point of view. One of the classic models by Fabrikant et al. [PODC'03] is the network creation game, where agents correspond to nodes in a network which buy incident edges for the price of alpha per edge to minimize their total distance to all other nodes. The model is well-studied but still has intriguing open problems. The most famous conjectures state that the price of anarchy is constant for all α and that for $\alpha \geq n$ all equilibrium networks are trees. We introduce a novel technique for analyzing stable networks for high edge-price alpha and employ it to improve on the best known bounds for both conjectures. In particular we show that for $\alpha > 4n - 13$ all equilibrium networks must be trees, which implies a constant price of anarchy for this range of alpha. Moreover, we also improve the constant upper bound on the price of anarchy for equilibrium trees.

- [33] Chauhan, A., Lenzner, P., Melnichenko, A., Molitor, L., [Selfish Network Creation with Non-Uniform Edge Cost](#). In: *Symposium on Algorithmic Game Theory (SAGT)*, pp. 160–172, 2017.

Network creation games investigate complex networks from a game-theoretic point of view. Based on the original model by Fabrikant et al. [PODC'03] many variants have been introduced. However, almost all versions have the drawback that edges are treated uniformly, i.e. every edge has the same cost and that this common parameter heavily influences the outcomes and the analysis of these games. We propose and analyze simple and natural parameter-free network creation games with non-uniform edge cost. Our models are inspired by social networks where the cost of forming a link is proportional to the popularity of the targeted node. Besides results on the complexity of computing a best response and on various properties of the sequential versions, we show that the most general version of our model has constant Price of Anarchy. To the best of our knowledge, this is the first proof of a constant Price of Anarchy for any network creation game.

- [34] Friedrich, T., Ihde, S., Kefler, C., Lenzner, P., Neubert, S., Schumann, D., [Efficient Best Response Computation for Strategic Network Formation under Attack](#). In: *Symposium on Algorithmic Game Theory (SAGT) (2017)*, pp. 199–211, 2017.

Inspired by real world examples, e.g. the Internet, researchers have introduced an abundance of strategic games to study natural phenomena in networks. Unfortunately, almost all of these games have the conceptual drawback of being computationally intractable, i.e. computing a best response strategy or checking if an equilibrium is reached is NP-hard. Thus, a main challenge in the field is to find tractable realistic network formation models. We address this challenge by investigating a very recently introduced model by Goyal et al. [WINE'16] which focuses on robust networks in the presence of a strong adversary who attacks (and kills) nodes in the network and lets this attack spread virus-like to neighboring nodes and their neighbors. Our main result is to establish that this natural model is one of the few exceptions which are both realistic and computationally tractable. In particular, we answer an open question of Goyal et al. by providing an efficient algorithm for computing a best response strategy, which implies that deciding whether the game has reached a Nash equilibrium can be done efficiently as well. Our algorithm essentially solves the problem of computing a minimal connection to a network which maximizes the reachability while hedging against severe attacks on the network infrastructure and may thus be of independent interest.

- [35] Friedrich, T., Ihde, S., Kefler, C., Lenzner, P., Neubert, S., Schumann, D., [Brief Announcement: Efficient Best Response Computation for Strategic Network Formation under Attack](#). In: *Symposium on Parallelism in Algorithms and Architectures (SPAA)*, pp. 321–323, 2017.

Inspired by real world examples, e.g. the Internet, researchers have introduced an abundance of strategic games to study natural phenomena in networks. Unfortunately, almost all of these games have the conceptual drawback of being computationally intractable, i.e. computing a best response strategy or checking if an equilibrium is reached is NP-hard. Thus, a main challenge in the field is to find tractable realistic network formation models. We address this challenge by establishing that the recently introduced model by Goyal et al. [WINE'16], which focuses on robust networks in the presence of a strong adversary, is a rare exception which is both realistic and computationally tractable. In particular, we sketch an efficient algorithm for computing a best response strategy, which implies that deciding whether the game has reached a Nash equilibrium can be done efficiently as well. Our algorithm essentially solves the problem of computing a minimal connection to a network which maximizes the reachability while hedging against severe attacks on the network infrastructure.

- [36] Chauhan, A., Lenzner, P., Melnichenko, A., Münn, M., [On Selfish Creation of Robust Networks](#). In: *Symposium on Algorithmic Game Theory (SAGT)*, pp. 141–152, 2016.

Robustness is one of the key properties of nowadays networks. However, robustness cannot be simply enforced by design or regulation since many important networks, most prominently the Internet, are not created and controlled by a central authority. Instead, Internet-like networks emerge from strategic decisions of many selfish agents. Interestingly, although lacking a coordinating authority, such naturally grown networks are surprisingly robust while at the same time having desirable properties like a small diameter. To investigate this phenomenon we present the first simple model for selfish network creation which explicitly incorporates agents striving for a central position in the network while at the same time protecting themselves against random edge-failure. We show that networks in our model are diverse and we prove the versatility of our model by adapting various properties and techniques from the non-robust versions which we then use for establishing bounds on the Price of Anarchy. Moreover, we analyze the computational hardness of finding best possible strategies and investigate the game dynamics of our model.

- [37] Cord-Landwehr, A., Lenzner, P., [Network Creation Games: Think Global - Act Local](#). In: *Mathematical Foundations of Computer Science (MFCS)*, pp. 248–260, 2015.

We investigate a non-cooperative game-theoretic model for the formation of communication networks by selfish agents. Each agent aims for a central position at minimum cost for creating edges. In particular, the general model (Fabrikant et al., PODC'03) became popular for studying the structure of the Internet or social networks. Despite its significance, locality in this game was first studied only recently (Bilo et al., SPAA'14), where a worst case locality model was presented, which came with a high efficiency loss in terms of quality of equilibria. Our main contribution is a new and more optimistic view on locality: agents are limited in their knowledge and actions to their local view ranges, but can probe different strategies and finally choose the best. We study the influence of our locality notion on the hardness of computing best responses, convergence to equilibria, and quality of equilibria. Moreover, we compare the strength of local versus non-local strategy changes. Our results address the gap between the original model and the worst case locality variant. On the bright side, our efficiency results are in line with observations from the original model, yet we have a non-constant lower bound on the Price of Anarchy.

- [38] Kawald, B., Lenzner, P., [On dynamics in selfish network creation](#). In: *Symposium on Parallelism in Algorithms and Architectures (SPAA)*, pp. 83–92, 2013.

We consider the dynamic behavior of several variants of the Network Creation Game, introduced by Fabrikant et al. [PODC'03]. Equilibrium networks in these models have desirable properties like low social cost and small diameter, which makes them attractive for the decentralized creation of overlay-networks. Unfortunately, due to the non-constructiveness of the Nash equilibrium, no distributed algorithm for finding such networks is known. We treat these games as sequential-move games and analyze if (uncoordinated) selfish play eventually converges to an equilibrium. Thus, we shed light on one of the most natural algorithms for this problem: distributed local search, where in each step some agent performs a myopic selfish improving move. We show that fast convergence is guaranteed for all versions of Swap Games, introduced by Alon et al. [SPAA'10], if the initial network is a tree. Furthermore, we prove that this process can be sped up to an almost optimal number of moves by employing a very natural move policy. Unfortunately, these positive results are no longer true if the initial network has cycles and we show the surprising result that even one non-tree edge suffices to destroy the convergence guarantee. This answers an open problem from Ehsani et al. [SPAA'11] in the negative. Moreover, we show that on non-tree networks no move policy can enforce convergence. We extend our negative results to the well-studied original version, where agents are allowed to buy and delete edges as well. For this model we prove that there is no convergence guarantee - even if all agents play optimally. Even worse, if played on a non-complete host-graph, then there are instances where no sequence of improving moves leads to a stable network. Furthermore, we analyze whether cost-sharing has positive impact on the convergence behavior. For this we consider a version by Corbo and Parkes [PODC'05] where bilateral consent is needed for the creation of an edge and where edge-costs are shared among the involved agents. We show that employing such a cost-sharing rule yields even worse dynamic behavior..

- [39] Lenzner, P. [Greedy Selfish Network Creation](#). In: *Web and Internet Economics (WINE)*, pp. 142–155, 2012.

We introduce and analyze greedy equilibria (GE) for the well-known model of selfish network creation by Fabrikant et al. [PODC'03]. GE are interesting for two reasons: (1) they model outcomes found by agents which prefer smooth adaptations over radical strategy-changes, (2) GE are outcomes found by agents which do not have enough computational resources to play optimally. In the model of Fabrikant et al. agents correspond to Internet Service Providers which buy network links to improve their quality of network usage. It is known that computing a best response in this model is NP-hard. Hence, poly-time agents are likely not to play optimally. But how good are networks created by such agents? We answer this question for very simple agents. Quite surprisingly, naive greedy play suffices to create remarkably stable networks. Specifically, we show that in the SUM version, where agents attempt to minimize their average distance to all other agents, GE capture Nash equilibria (NE) on trees and that any GE is in 3-approximate NE on general networks. For the latter we also provide a lower bound of $3/2$ on the approximation ratio. For the MAX version, where agents attempt to minimize their maximum distance, we show that any GE-star is in 2-approximate NE and any GE-tree having larger diameter is in $6/5$ -approximate NE. Both bounds are tight. We contrast these positive results by providing a linear lower bound on the approximation ratio for the MAX version on general networks in GE. This result implies a locality gap of $\Omega(n)$ for the metric min-max facility location problem, where n is the number of clients.

- [40] Lenzner, P. [On Dynamics in Basic Network Creation Games](#). In: *Symposium on Algorithmic Game Theory (SAGT)*, pp. 254–265, 2011.

We initiate the study of game dynamics in the Sum Basic Network Creation Game, which was recently introduced by Alon et al. [SPAA'10]. In this game players are associated to vertices in a graph and are allowed to "swap" edges, that is to remove an incident edge and insert a new incident edge. By performing such moves, every player tries to minimize her connection cost, which is the sum of distances to all other vertices. When played on a tree, we prove that this game admits an ordinal potential function, which implies guaranteed convergence to a pure Nash Equilibrium. We show a cubic upper bound on the number of steps needed for any improving response dynamic to converge to a stable tree and propose and analyse a best response dynamic, where the players having the highest cost are allowed to move. For this dynamic we show an almost tight linear upper bound for the convergence speed. Furthermore, we contrast these positive results by showing that, when played on general graphs, this game allows best response cycles. This implies that there cannot exist an ordinal potential function and that fundamentally different techniques are required for analysing this case. For computing a best response we show a similar contrast: On the one hand we give a linear-time algorithm for computing a best response on trees even if players are allowed to swap multiple edges at a time. On the other hand we prove that this task is NP-hard even on simple general graphs, if more than one edge can be swapped at a time. The latter addresses a proposal by Alon et al..

- [41] Antoniadis, A., Hüffner, F., Lenzner, P., Moldenhauer, C., Souza, A., [Balanced Interval Coloring](#). In: *Symposium on Theoretical Aspects of Computer Science (STACS)*, pp. 531–542, 2011.

We consider the discrepancy problem of coloring n intervals with k colors such that at each point on the line, the maximal difference between the number of intervals of any two colors is minimal. Somewhat surprisingly, a coloring with maximal difference at most one always exists. Furthermore, we give an algorithm with running time $O(n \log n + kn \log k)$ for its construction. This is in particular interesting because many known results for discrepancy problems are non-constructive. This problem naturally models a load balancing scenario, where n tasks with given start- and endtimes have to be distributed among k servers. Our results imply that this can be done ideally balanced. When generalizing to d -dimensional boxes (instead of intervals), a solution with difference at most one is not always possible. We show that for any $d \geq 2$ and any $k \geq 2$ it is NP-complete to decide if such a solution exists, which implies also NP-hardness of the respective minimization problem. In an online scenario, where intervals arrive over time and the color has to be decided upon arrival, the maximal difference in the size of color classes can become arbitrarily high for any online algorithm.

- [42] Albers, S., Lenzner, P., [On Approximate Nash Equilibria in Network Design](#). In: *Web and Internet Economics (WINE)*, pp. 14–25, 2010.

We study a basic network design game where n self-interested agents, each having individual connectivity requirements, wish to build a network by purchasing links from a given set of edges. A fundamental cost sharing mechanism is Shapley cost sharing that splits the cost of an edge in a fair manner among the agents using the edge. In this paper we investigate if an optimal minimum-cost network represents an attractive, relatively stable state that agents might want to purchase. We resort to the concept of α -approximate Nash equilibria. We prove that for single source games in undirected graphs, any optimal network represents an $H(n)$ -approximate Nash equilibrium, where $H(n)$ is the n -th Harmonic number. We show that this bound is tight. We extend the results to cooperative games, where agents may form coalitions, and to weighted games. In both cases we give tight or nearly tight lower and upper bounds on the stability of optimal solutions. Finally we show that in general source-sink games and in directed graphs, minimum-cost networks do not represent good states.