















problem size  $n$  is varied in  $[100, 1000]$  (with a step size of 25). In each tested setting, the run is replicated 100 times with different random seeds and the number of function evaluations, denoted as ‘# evaluations’, is reported as the run time. The result for  $k = 4$  is shown in Figure 1. Note that we also did the experiment with  $p_c = 0$  for the no-mechanism setting, e.g., comparing the EA with the GAs, however, the average run time for  $n = 100$  in this experiment is already  $2.28 \cdot 10^8$ , which cannot be displayed in the figure.

On average, the highest contribution to the reduction of the run time in order is fitness sharing, then convex hull maximisation, deterministic crowding, and, finally, duplicate elimination and minimisation have quite similar average run times. We also notice that the island model with  $\mu = 2$  requires approximately the same average numbers of evaluations as deterministic crowding. Overall, compared to the standard  $(\mu+1)$  GA, all the diversity mechanisms contribute to the reduction of the average run time, as well as to the stability of the result.

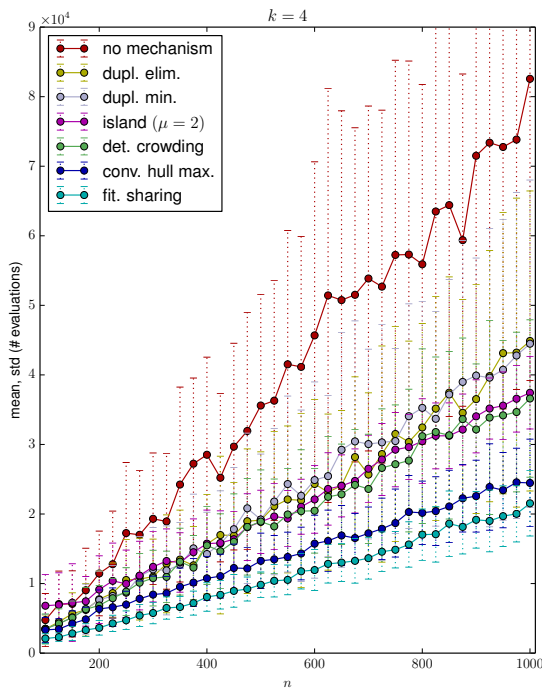


Figure 1: Performance of the diversity mechanisms.

## 7. CONCLUSION

We have considered the role of selection-based diversity mechanisms used together with crossover for escaping local optima. We prove rigorous upper bounds on the run time of the  $(\mu+1)$  GA for seven well-known diversity mechanisms optimising the  $\text{Jump}_k$  function. Our results reveal a qualitative difference in the ability of the different diversity mechanisms to escape local optima.

In contrast to previous theoretical work on crossover for  $\text{Jump}_k$ , our upper bounds do not rely on unreasonably small (e.g., vanishing with  $n$ ) crossover probabilities, but instead cover the more practical case of constant crossover probabilities. Furthermore, our proofs provide insight into the ways that diversity mechanisms, when applied as a tie-breaking rule in selection, can quickly spread the population out over the jump plateau in order to get enough diversity for

crossover to combine the correct solution components to escape the set of local optima.

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