STAIRS 2012
K. Kersting and M. Toussaint (Eds.)

doi:10.3233/978-1-61499-096-3-71

Deliberative Acceptability of Arguments

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Abstract. State of the art extension based argument acceptance is currently biased toward attacks: while the defending extension of an argument a is internally coherent, no such requirement is imposed on its attacking set. On the other hand, if we restrict ourselves only to conflict-free sets of attacking arguments, then we could have different attacking sets for a specified argument a (any two conflicting attackers of a must belong to different a's attacking sets). Having only one defending extension for all these attacking sets would contradict the deliberative nature of argumentation in the real world, where only the coherent sets of attacks matter and the defending sets of arguments depend on the former.

In this paper we introduce a new type of acceptability of an argument, in which its attacking and defending sets of arguments are uniformly treated. We call it *deliberative acceptance*, discuss how this and the classical acceptance notions interrelate and analyze its computational properties. In particular, we prove that the corresponding decision problem is Π_2^{P}-complete, but its restrictions on bipartite or co-chordal argumentation frameworks are in P.

Keywords. abstract argumentation, argument acceptability, computational properties of argumentation

1. Introduction

The notion of argument acceptance is central in argumentation. The graph-theoretic model of argumentation frameworks introduced by Dung [1] focuses on the manner in which a specified set A of abstract arguments interact via an attack binary relation D on A. If $(a,b) \in D$ (argument *a attacks* argument b) we have a conflict. A *conflict-free* set of arguments is a set $T \subseteq A$ such that there are no $a,b \in T$ with $(a,b) \in D$.

In this model, one of the main aim of argumentation is in deciding the status of some argument in presenting a justification for this. More precisely, the acceptability of an argument a is defined based on its membership in a set of arguments satisfying certain properties. A family $\mathbb{S} \subset 2^A$ of sets of arguments is defined (the predicate $S \in \mathbb{S}$ is called *semantics* in this context) and a is considered *acceptable* if there is $S \in \mathbb{S}$ such that $a \in S$ – *credulous acceptance* – or if $a \in S$ for all $S \in \mathbb{S}$ – *skeptical acceptance*. This kind of rationality (based on the possibility of extending the analyzed argument to a set of "collectively acceptable" arguments) is called *extension based semantics*. The *grounded, preferred* and *stable semantics* defined by Dung (see Section 2) formalizes different intuitions about which arguments to accept on the basis of a given framework. Discussions on how to select the appropriate semantics for a given application context or how to compare the different semantics for argumentation frameworks, have been the subject of several papers (see [2], [3], [4], [5], [6], etc.). The motivation of considering different semantics (*semi-stable* [7], *prudent* [8], *ideal* [9], *SCC-recursive* [10], *evidence-based* [11], etc.) is given by the need to formalize "everyday reasoning" in order to design mechanisms expressing a "legitimate" argumentation in support of an argument.

The above sets of collectively acceptable arguments have at least the properties of being conflict-free (that is, they are internally logically coherent) and defend themselves from any external attack (that is, they survive the attacks together). This type of argument acceptance is biased toward attacks: while the defending extension of an argument a is internally coherent, no such requirement is imposed on its attacking set. On the other hand, if we restrict ourselves only to conflict-free sets of attacking arguments, then we could have different attacking sets for a specified argument a (any two conflicting attackers of a must belong to different a's attacking sets). In many cases in the real world, when argument a is attacked by a coherent set of arguments T, a is defended by giving a coherent set S including a, which defends against T.

In this paper we follow this intuition and introduce a notion of acceptability at the level of justification states of arguments rather than of extensions. We call it *deliberative acceptability* due to the uniform treatment of both attacking and defending sets in the definition of the acceptability of an argument. More precisely, an argument a is *deliberatively acceptable* in a given argumentation framework if, for each conflict-free set of arguments attacking a, there is a conflict-free set of arguments containing a and defending itself against the former set. We analyze this type of acceptability and investigate it from a complexity point of view. In Proposition 8, we prove that if an argument is credulously grounded, preferred or stable accepted then it is deliberatively accepted, but the converse is not true. This is illustrated in Figure 1.

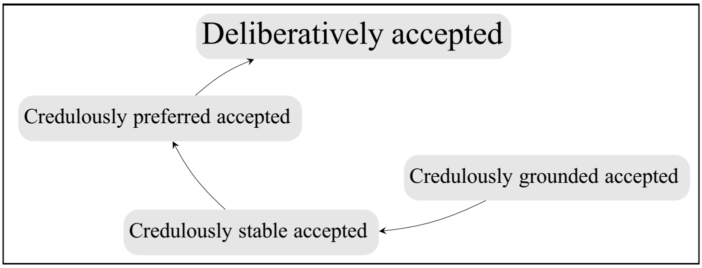

Figure 1. Acceptability implications (edges implied by transitivity are omitted).

We give a sufficient graph theoretical condition in which a deliberatively accepted argument belongs to an admissible set (see Proposition 9). Also, we show that in bipartite[1] or symmetric argumentation framework the deliberative acceptability is equivalent to credulous preferred acceptability (Proposition 10). In Proposition 12, we prove that the problem of deciding if an argument is deliberatively accepted in a given argumentation framework is Π_2^P-complete but its restrictions on bipartite or co-chordal argumentation frameworks are polynomial time solvable (Propositions 13 and 14).

The remainder of the paper is organized as follows. After reviewing the necessary basic concepts in Section 2, the notion of *deliberative acceptance* is introduced and its basic properties are studied in Section 3. We close this section with three examples con-

[1]A graph is called *bipartite* iff it has no cycle of odd length.

cerning the relation of the deliberative acceptance with the extension semantics which are not admissibility based. In Section 4, we proved that the corresponding decision problem is Π_2^P-complete and two restrictions under which it is in P are identified. Finally, Section 5 concludes the paper and discusses future work.

2. Dung's Theory of Argumentation and Extensions

In this section we present the basic concepts used for defining usual semantics in abstract argumentation frameworks introduced by Dung in 1995, [1]. In such argumentation frameworks the focus is only on the *defeat relation* between arguments from an already established finite set A of arguments.

Definition 1 *An* **Argumentation Framework** *is a digraph* $AF = (A,D)$*, where vertices from* A *are called* **arguments***, and if* $(a,b) \in D$ *then* **argument** a **defeats (attacks) argument** b**.**

We suppose that the digraph AF has no loops, that is, there are no self-attacking arguments in A. We also consider only finite digraphs, that is, the set A is finite and non-empty.

Let $AF = (A,D)$ be an argumentation framework. For each $a \in A$ let $a^+ = \{b \in A |\ (a,b) \in D\}$ be the set of all arguments *attacked* by a, and $a^- = \{b \in A |\ (b,a) \in D\}$ be the set of all arguments *attacking* a. These notations can be extended to sets of arguments. The set of all arguments *attacked by* (the arguments in) $S \subseteq A$ is $S^+ = \bigcup_{a \in S} a^+$, and the set of all arguments *attacking* (the arguments in) S is $S^- = \bigcup_{a \in S} a^-$.

Definition 2 *Let* $S \subseteq A$ *be a set of arguments.*

- *S is called* **conflict-free** *if* $S \cap S^+ = \emptyset$ *(i.e. there are no attacking arguments in S).*
- *S* **defends** *an argument* $a \in A$ *if* $a^- \subseteq S^+$ *(i.e. any a's attacker is attacked by an argument in S). The set of* ***all arguments defended by a set*** *S of arguments is denoted by* $F(S)$*.*
- *S is* **admissible** *if it is conflict-free and* $S \subseteq F(S)$ *(i.e. defends its elements).*
- *S is a* **complete extension** *if it is conflict-free and* $S = F(S)$ *(i.e. the set of all elements defended by S is exactly S).*
- *S is a* **preferred extension** *if it is a maximal (w.r.t. $\subseteq$) complete extension.*
- *S is a* **grounded extension** *if it is a minimal (w.r.t. $\subseteq$) complete extension.*
- *S is a* **stable extension** *if it is conflict-free and* $S^+ = A - S$*.*

It is not difficult to see that any admissible set is contained in a preferred extension, which exists in any argumentation framework; the preferred extension is unique if the argumentation framework has no directed cycle of even length ([4], [10]).

The grounded extension exists and it is unique in any argumentation framework. It can be constructed by considering all non-attacked arguments, deleting these arguments and those attacked by them from the digraph, and repeating these two steps for the digraph obtained until no node remains. Clearly, the grounded extension is a natural extension for an argumentation framework, but it could be empty (if in the argumentation framework each argument has at least one attacker).

Definition 3 *Let $AF = (A,D)$ be an argumentation framework. An* **extension-based acceptability semantics** *is any predicate $\sigma : 2^A \to \{true, false\}$. The set of all extensions under σ, or the set of all σ-extensions, is $\mathcal{E}_\sigma = \{S \subseteq A \,|\, \sigma(S) = true\}$.*

Definition 4 *Let $AF = (A,D)$ be an argumentation framework, $a \in A$ and σ a semantics. a is* **credulously accepted** *under σ if there is $S \in \mathcal{E}_\sigma$ such that $a \in S$. a is* **sceptically accepted** *under σ if $a \in S$ for all $S \in \mathcal{E}_\sigma$.*

Example 5 *Let us consider the argumentation framework in the Figure 2 below.*

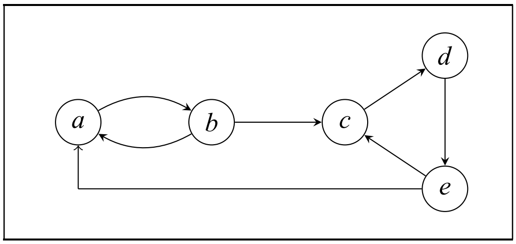

Figure 2. Argumentation Framework in Example 1

Since each argument has at least one attacker, the grounded extension is empty. The conflict-free sets containing the argument a are $\{a\}$, $\{a,c\}$, and $\{a,d\}$. Since a cannot defend against the attacker e, $\{a\}$ is not admissible. The set $\{a,c\}$ cannot defend against the attacker e, and the set $\{a,d\}$ cannot defend against the attacker c. It follows that a does not belong to a preferred extension. The unique preferred extension is $\{b,d\}$ (its attackers are defeated by b). Therefore we could accept b and not a, because b can be extended to a maximal conflict-free set of arguments $\{b,d\}$ defending itself from all attacks.

Despite of the fact that a is not credulously preferred accepted, its "acceptability" could be argued as follows. The conflict-free sets attacking a are: $\{b\}$, $\{b,d\}$ and $\{b,e\}$. Clearly, a defends itself against $\{b\}$. Against $\{b,d\}$, we can defend a by considering the conflict-free set $\{a,c\}$, that is choosing a coherent sets of arguments on the same idea induced by the attacking set $\{b,d\}$. The set $\{a,c\}$ is not appropriate for the attacking set $\{b,e\}$ but, in this case, $\{a,d\}$ does the job. Hence we can defend a against all conflict-free sets of arguments attacking it, therefore we can "accept" a.

We will define and analyze this type of acceptability in the next section.

3. Deliberative Acceptability of Arguments

Definition 6 *Let $AF = (A,D)$ an argumentation framework. An argument $a \in A$ is* **deliberative acceptable** *if for any conflict-free set T attacking a (i.e. $T \cap a^- \neq \emptyset$), there is a conflict-free set $S \subset A$ such that $a \in S$ and S defends itself against T (i.e. $T \cap S^- \subseteq S^+$).*

This type of acceptability is closer to the intuition about real life debate-type argumentation. If the argument a is attacked by a conflict-free set T of arguments, there is a conflict-free set of arguments S containing a, depending on T such that S defends a. S also defends its arguments that are attacked by T. Figure 3 illustrates two conflict-free sets T_i attacking a and the corresponding two answers to these attacks, two conflict-free sets S_i containing a. Each attack of a from T_i is counterattacked by an argument from S_i. If T_i attacks also this defender, then S_i has also a counterattack to this. And so on.

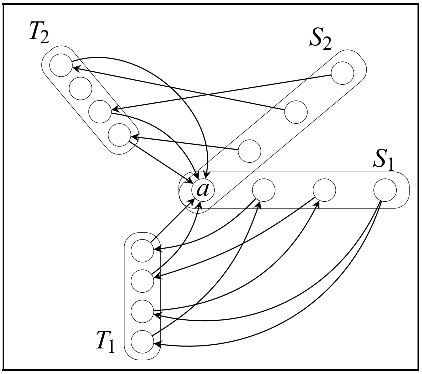

Figure 3. Deliberative Acceptability Example. Two conflict-free sets T_1 and T_2 attacking a and the corresponding defending conflict-free sets S_1 and S_2.

Note that we assumed that there are not self-attacking arguments in the argumentation framework. If a would be a self-attacking argument, then a is not contained in any conflict-free set. Hence, if a is attacked by some other argument then a cannot be deliberatively accepted. Moreover, since a does not belong to any conflict-free set, the deliberative acceptance of other arguments is not influenced. Thus, without loss of generality, we restrict our attention to argumentation frameworks without self-attacking arguments.

It is possible to introduce a weaker form of the deliberative acceptance by considering in the above definition only the conflict-free sets T contained in a^-. This could be too tolerant as the following example shows.

Example 7 *Let us consider the argumentation framework in the Figure 4 below.*

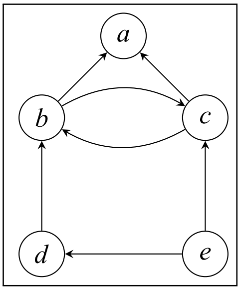

Figure 4. Argument a is not deliberative acceptable, despite it can be defended against conflict-free sets contained in a^-.

The grounded extension is $\{e,b\}$. *It follows that the argument a is not credulously accepted in any classical extension semantics. The conflict-free sets contained in* a^- *are* $\{b\}$ *and* $\{c\}$ *which can be defeated by the conflict-free sets containing a:* $\{d,a\}$, *respectively, $\{e,a\}$. If we consider a (weak) deliberative acceptable this will correspond to a superficial analysis. Indeed, there are also two conflict-free sets attacking a, namely $\{c,d\}$ and $\{b,e\}$. The set $\{e,a\}$ defeats $\{c,d\}$, but $\{e,b\}$ is a conflict-free set attacking a, which could not be defended. It follows that the argument a is not deliberatively accepted.*

The following proposition shows that deliberative acceptance is strictly more liberal than credulous acceptance with respect to the classical extension-based semantics.

Proposition 8 *Let $AF = (A,D)$ be an argumentation framework and $a \in A$. If a is σ-credulously accepted for $\sigma \in \{$ grounded, preferred, stable$\}$, then a is deliberatively accepted. For each the above semantics σ, there are argumentation frameworks in which a deliberatively accepted argument is not σ-credulously accepted.*

Proof. If $a \in A$ is σ-credulously accepted for $\sigma \in \{$grounded, preferred, stable$\}$, then there is an admissible set S_0 containing a. It follows that for any $b \in S_0^-$ there is $s \in S_0$ such that $(s,b) \in D$. Let T be a conflict-free set of arguments attacking a. Each argument $b \in T \cap S_0^-$ is attacked by S_0. Hence $T \cap S_0^- \subseteq S_0^+$, and therefore a is deliberative accepted.
In the argumentation framework in Figure 2 the grounded extension is $\emptyset$. Hence a is not grounded-credulous accepted. However, as we argued in the end of Section 2, a is deliberative accepted. Also, a is not preferred accepted because the unique preferred extension $\{b,d\}$ does not contain a. Since any stable extension is a preferred extension, it follows that a is also not stable-credulously accepted. $\square$

Depending on the combinatorial structure of the argumentation framework, the deliberative acceptance could agree to that based on extensions. The following proposition gives an easy sufficient condition for this.

Proposition 9 *Let $AF = (A,D)$ be an argumentation framework and $a \in A$. If AF does not contain the induced subdigraphs F_1 and F_2 in Figure 5 (dotted directed edges are optional), then, if a is deliberatively accepted, it follows that a is credulously preferred accepted.*

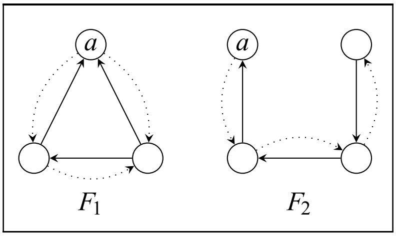

Figure 5. Forbidden induced subdigraphs F_1 and F_2.

Proof. Since F_1 is forbidden, it follows that a^- is conflict-free (if there are b,c such that $(b,a),(b,c),(c,a) \in D$, then $\{a,b,c\}$ induces F_1). Taking $T_0 = a^-$, since a is deliberatively acceptable, there is a conflict-free set S such that $a \in S$ and $T_0 \cap S^- \subseteq S^+$. Let S_{T_0} be a minimal set S with these properties. We prove that S_{T_0} is admissible. Indeed, let b be an argument attacking S_{T_0}. If $(b,a) \in D$, then $b \in T_0$, therefore there is $a' \in S_{T_0}$ such that $(a',b) \in D$. Hence S_{T_0} defends itself against b. If $b \notin T_0$, then either $(a,b) \in D$ or $\{a,b\}$ is conflict-free. If $(a,b) \in D$, then, since $a \in S_{T_0}$, it follows that S_{T_0} defends itself against b. If $\{a,b\}$ is conflict-free, then since b attacks S_{T_0}, there is $c \in S_{T_0}$, $c \neq a$ such that $(b,c) \in D$. By the minimality of S_{T_0}, there is $d \in T_0$ such that $(c,d) \in D$. But then the subdigraph induced in AF by $\{a,b,c,d\}$ is F_2 in Figure 5, which yields a contradiction. It follows that S_{T_0} is admissible and hence can be extended to a maximal complete extension containing a, that is a is credulously preferred accepted. □

Using similar arguments as in the above proof, it is not difficult to show that if AF is a *directed acyclic graph* (DAG) or if the underlying undirected graph of AF is bipartite, then an argument a is deliberatively accepted if and only if a is credulously preferred accepted. However, for bipartite graphs we can do better as follows.

Let $G = (A,E)$ be the underlying undirected graph of AF ($\{a,b\} \in E$ if and only if $(a,b) \in D$ or $(b,a) \in D$). Since G is bipartite, then A can be partitioned $A = U \cup V$, $U,V \neq \emptyset$, $U \cap V = \emptyset$, and if $\{a,b\} \in E$ then $|\{a,b\} \cap V| = 1$ and $|\{a,b\} \cap U| = 1$.

In [12], Dunne proved that the following algorithm applied to a bipartite argumentation framework $AF = (A = U \cup V, D)$ (below, $[X]_{AF}$ denotes the subdigraph induced by $X \subseteq A$ in AF: $[X]_{AF} = (X, D \cap X \times X)$)

> **input** $AF = (U \cup V, D)$
> **while** $\exists v \in V - U^+$ s.t. $v^+ \cap U \neq \emptyset$ **do**
> $\quad AF := [(U - v^+) \cup V]_{AF}$
> **return** U

returns a final set U, denoted U_0, which satisfies: U_0 is conflict-free (is a subset of U), and $U_0^- \subseteq U_0^+$ (in the given argumentation framework AF). Similarly, the algorithm:

> **input** $AF = (U \cup V, D)$
> **while** $\exists u \in U - V^+$ s.t. $u^+ \cap V \neq \emptyset$ **do**
> $\quad AF := [U \cup (V - u^+)]_{AF}$
> **return** V

returns a final set V, denoted V_0, which satisfies: V_0 is conflict-free (is a subset of V), and $V_0^- \subseteq V_0^+$. Moreover, $a \in A$ is credulous preferred accepted if and only if $a \in U_0$ or $a \in V_0$ (Dunne, [12]). It is not difficult to prove that the following proposition holds.

Proposition 10 *Let $AF = (A,D)$ be an argumentation framework and $a \in A$. If the underlying undirected graph of AF is bipartite with bipartition $A = U \cup V$, and U_0 and V_0 are the sets of arguments constructed by the Dunne's algorithm above, then a is deliberatively accepted if and only if $a \in U_0 \cup V_0$.*

Proof. The "only if" part follows from Proposition 8 and Dunne's characterization of credulous preferred arguments in bipartite argumentation frameworks [12].

To prove the "if" part, suppose that there is $a \in U - U_0$ deliberative acceptable (similarly, if $a \in V - V_0$). It follows that there are (distinct) vertices $v_1, \ldots, v_k \in V$ such that v_i is used in the i^{th} "while" iteration of the algorithm above, to delete vertices from U and v_k is the first such vertex attacking a ($a \in v_k^+$). Since a is deliberative acceptable, for the conflict-free set $T = \{v_1, \ldots, v_k\}$ attacking a, there is a conflict-free set S such that $a \in S$ and $S^- \cap T \subseteq S^+$. Since v_1 is not attacked in the initial framework, it follows that $v_1 \notin S^-$.

Since v_2 is not attacked in the second framework, it follows that $v_2 \notin S^-$. Inductively, we obtain that $v_k \notin S^-$, contradicting the hypothesis $a \in v_k^+$. The contradiction obtained ends the proof of the "if" part. □

We note that if the argumentation framework is symmetric (that is, AF is obtained from an undirected graph by replacing each undirected edge $\{a,b\}$ by the pair (a,b) and (b,a) of directed edges), then each argument is deliberative acceptable. This is obvious since, in this case, a set of arguments is admissible if and only if is conflict-free, [8].

We close this section with three examples concerning the relation of the deliberative acceptance with the extension semantics which are not admissibility based: stage semantics [13], and cf2 semantics [14]. One of the advantages of these two semantics is that odd and even directed cycles are treated equally.

A stage extension in $AF = (A,D)$ is a conflict-free set $S \subseteq A$ where $S \cup S^+$ is maximal (w.r.t. set inclusion) among all conflict-free sets in AF. A recursive definition for the cf2 extensions is: a set $S \subseteq A$ is a cf2-extension in $AF = (A,D)$ if

- AF is strongly connected and S is a maximal (w.r.t. set inclusion) conflict-free set in AF, or
- AF is not strongly connected and for any strongly connected component C of AF, $S \cap C$ is a cf2 extension in the argumentation framework $AF[C-(S-C)^+]$ induced by $C-(S-C)^+$ in AF.

Example 11 *In the argumentation framework AF_1 in the Figure 6 below, the stage extensions and the cf2 extensions are $\{a,c,e\}$ and $\{b,d,f\}$. Hence each argument is accepted under stage and cf2 semantics. The same conclusion is obtained with the deliberative acceptance. Indeed, these two extensions are also admissibility extensions and therefore each argument is deliberatively accepted by Proposition 8.*

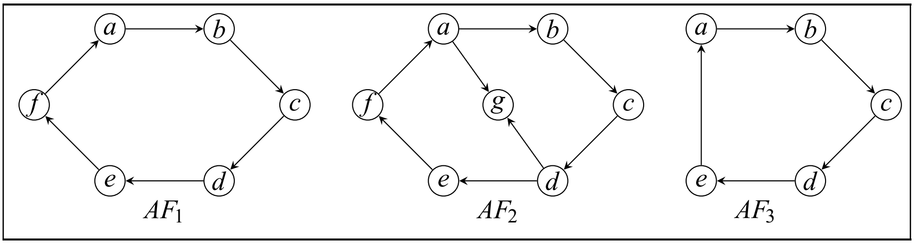

Figure 6. Three argumentation frameworks with different deliberative acceptance conclusions versus stage and cf2 semantics.

In the argumentation framework AF_2, the stage extensions and the cf2 extensions are also $\{a,c,e\}$ and $\{b,d,f\}$ (which are also stable extensions). Hence the argument g is not accepted under stage and cf2 semantics. The conflict-free set $T = \{a,e\}$ attacks g. A conflict free set S containing g and defending itself against T must contain d (the only attacker of e) which is not possible (d attacks g). It follows that g is not deliberatively accepted. In the argumentation framework AF_3, the stage extensions and the cf2 extensions are $\{a,c\}$, $\{a,d\}$, $\{b,d\}$, $\{b,e\}$. Hence each argument is accepted under stage and cf2 semantics. Let $T = \{e,c\}$ be a conflict-free set attacking a. As above, a conflict-free S containing a and defending itself against T must contain b, a contradiction. Hence a is not deliberatively accepted.

4. Complexity

Let us consider the following decision problem:
Deliberative Acceptability
Instance: $AF = (A,D)$ argumentation framework, $a \in A$.
Question: Is a deliberatively acceptable?

The complexity class Π_2^{P} comprises those problems decidable by co-NP computations given (unit cost) access to an NP complete oracle. Alternatively, Π_2^{P} can be viewed as the class of languages L whose membership is certified by a polynomial-time testable ternary relation $R_L \subseteq W \times X \times Y$: there is a polynomial p such that, for all w, $w \in L$ if and only if $(\forall x \in X : |x| \leq p(|w|))(\exists y \in Y : |y| \leq p(|w|))\ (w,x,y) \in R_L$.

Proposition 12 **Deliberative acceptability** *is* Π_2^{P}*-complete.*

Proof. It is easy to see that **Deliberative acceptability** is in Π_2^{P}, since it corresponds to the language $L = \{w|\forall x \exists y R(w,x,y)\}$, where w encodes an instance (AF,a) of the problem and $(w,x,y) \in R$ if and only if x encodes a conflict-free set T attacking a, and y encodes a conflict-free set S containing a such that S defends itself against T.

We prove Π_2^{P}-hardness for **Deliberative acceptability** by a reduction from the decision problem $\forall\exists$SAT.

An instance of $\forall\exists$SAT is a formula $F' = \forall x_1 \ldots \forall x_n \exists y_1 \ldots \exists y_n F(x_1,\ldots,x_n,y_1,\ldots,y_n)$, where $F(x_1,\ldots,x_n,y_1,\ldots,y_n)$ is a CNF formula over the disjoint sets of variables $X = \{x_1,\ldots,x_n\}$ and $Y = \{y_1,\ldots,y_n\}$. The instance $F' = \forall X \exists Y F(X,Y)$ is accepted if and only if for any truth assignment α_X of the variables in X, there is a truth assignment α_Y of the variables in Y such that (α_X, α_Y) satisfies the formula $F(X,Y)$. It is well known that $\forall\exists$SAT is Π_2^p-complete [15,16].

We will construct in polynomial time an argumentation framework $AF_{F'}$ for each instance F' of $\forall\exists$SAT. Let $F' = \forall X \exists Y F(X,Y)$ and $F(X,Y) = C_1 \wedge \ldots \wedge C_m$, where each clause C_i is a disjunction of literals. A literal is a variable $x_i \in X = \{x_1,\ldots,x_n\}$, $y_i \in Y = \{y_1,\ldots,y_n\}$, or their negations $\bar{x}_i \in \overline{X} = \{\overline{x_1},\ldots,\bar{x}_n\}$ and $\bar{y}_i \in \overline{Y} = \{\overline{y_1},\ldots,\bar{y}_n\}$.

The argumentation framework associated to F' is $AF_{F'} = (A,D)$, where:

- $A = \{F\} \cup \{C_1,\ldots,C_m\} \cup X \cup \overline{X} \cup Y \cup \overline{Y}$, and
- $D = \cup_{i=1}^{m}\{(C_i,F)\} \bigcup$
$\cup_{i=1}^{m} \cup_{j=1}^{n} \{(x_j,C_i)|x_j \text{ occurs in } C_i\} \bigcup \cup_{i=1}^{m} \cup_{j=1}^{n} \{(\bar{x}_j,C_i)|\bar{x}_j \text{ occurs in } C_i\} \bigcup$
$\cup_{i=1}^{m} \cup_{j=1}^{n} \{(y_j,C_i)|y_j \text{ occurs in } C_i\} \bigcup \cup_{i=1}^{m} \cup_{j=1}^{n} \{(\bar{y}_j,C_i)|\bar{y}_j \text{ occurs in } C_i\} \bigcup$
$\cup_{j=1}^{n} \{(x_j,\bar{x}_j),(\bar{x}_j,x_j),(y_j,\bar{y}_j),(\bar{y}_j,y_j)\} \bigcup$
$\cup_{i=1}^{n} \cup_{j=1}^{n} \{(x_i,y_j),(x_i,\bar{y}_j)\} \bigcup \cup_{i=1}^{n} \cup_{j=1}^{n} \{(y_i,x_j),(y_i,\bar{x}_j)\} \bigcup$
$\cup_{i=1}^{n} \cup_{j=1}^{m} \{(C_j,y_i),(C_j,\bar{y}_i)\} \bigcup$
$\cup_{i=1}^{n} \{(F,x_i),(F,\bar{x}_i)\}$.

Clearly, $AF_{F'}$ can be constructed in polynomial time from F'. Its structure is visualized in Figure 7 below. Note that the only conflict-free sets of arguments containing F are subsets of the form $\{F\} \cup S$, where $S \subset Y \cup \overline{Y}$. Also, the only conflict-free sets of arguments meeting C are of the form $C' \cup T$, where $\emptyset \neq C' \subseteq C$ and $T \subset X \cup \overline{X}$.

We prove that $F' = \forall X \exists Y F(X,Y)$ is an accepted instance of $\forall\exists$SAT if and only if F is deliberatively accepted in $AF_{F'}$.

Suppose that F is deliberatively accepted in $AF_{F'}$. Let α_X be any truth assignment for the variable in X. If all clauses from C are satisfied by α_X then (α_X, α_Y) is a satisfying

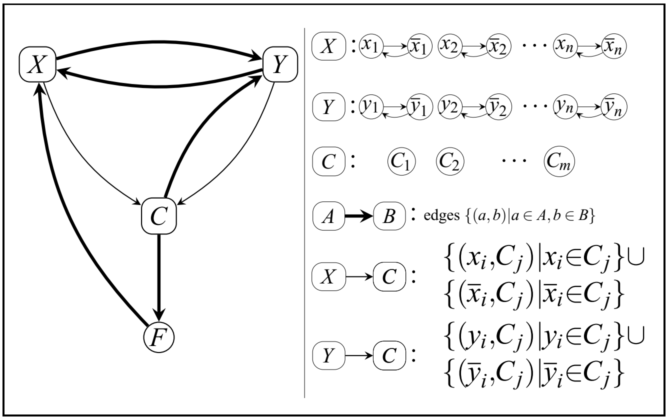

Figure 7. The argumentation framework $AF_{F'}$ associated to instance $F' = \forall X \exists Y F(X,Y)$.

assignment for $F(X,Y)$, for any α_Y. Suppose that there is a nonempty subset $C' \subseteq C$ such that no clause C_i of C' is satisfied by α_X. This means that for every literal $l \in (X \cup \overline{X}) \cap C_i$, we have $\alpha(l) = false$. It follows that $T = \{l \in X \cup \overline{X} | \alpha(l) = true\} \cup C'$ is a conflict-free set in $AF_{F'}$ attacking the argument F (by all arguments in C'). Since F is deliberatively accepted, there is a conflict-free set of argument S such that $F \in S$ and $T \cap S^- \subseteq S^+$. It follows that $T - \{F\} \subseteq Y \cup \overline{Y}$ and for each $C_i \in C'$ there is $l \in Y \cup \overline{Y}$ such that $(l, C_i) \in D$, that is $l \in C_i$. If we consider α_Y, the assignment with $\alpha_Y(l) = true$ for all these literals l, we obtain that (α_X, α_Y) is a satisfying assignment for $F(X,Y)$. Hence F' is a positive instance of ∀∃SAT.

Conversely, let $F' = \forall X \exists Y F(X,Y)$ be a positive instance of ∀∃SAT. Let T be a conflict-free set of arguments attacking F (that is, $C' = T \cap C \neq \emptyset$). Let α_X a truth assignment for variables in X such that for each $l \in (X \cup \overline{X}) \cap T$ we have $\alpha_X(l) = false$ (such assignment exists since T is conflict-free). Since F' is a positive instance of ∀∃SAT, there is a truth assignment α_Y for the variables in Y such that for each $C_i \in C'$ there is a literal $l_i \in (Y \cup \overline{Y}) \cap C_i$ such that $\alpha_Y(l_i) = true$. Taking S the set of arguments containing F and all such literals l_i, we obtain a conflict-free set of arguments with the property that $T \cap S^- \subseteq S^+$. It follows that F is deliberatively accepted in $AF_{F'}$. □

It is interesting to identify graph-theoretic constraints for an argumentation framework under which the problem **Deliberative acceptability** becomes polynomial-time solvable. A first such constraint follows from Proposition 10 and the polynomial runtime of Dunne's algorithm.

Proposition 13 *If the underlying undirected graph associated to the argumentation framework $AF = (A,D)$ is bipartite, then* **Deliberative acceptability** *is in* P.

A more interesting restriction of the **Deliberative acceptability** problem can be obtained by imposing that the underlying undirected graph associated to the argumentation framework to be co-chordal. A chordal (triangulated) graph is an undirected graph which does not contain as an induced subgraph the cycle graph C_k, for any $k \geq 4$ (equivalently, in such a graph every cycle of length at least 4 has a chord). The complement of a chordal graph is a co-chordal graph. Chordal graphs can be recognized in $O(n+m)$ time, where n is its number of vertices and m is its number of edges [17,18]. Also, the number of cliques (maximal complete subgraphs) of a chordal graph is $O(n)$ and all these cliques can be found in $O(n+m)$, using *Maximum Cardinality Search*, [18]. It follows that in a co-chordal graph the number of maximal independent sets of vertices (maximal stable sets) is $O(n)$ and all these maximal stable sets can be found in linear time. Hence, for the argumentation frameworks with an underlying co-chordal graph, the number of maximal conflict-free sets is linear.

Proposition 14 *If the underlying undirected graph associated to the argumentation framework $AF = (A,D)$ is a co-chordal graph, then* **Deliberative acceptability** *is in* P.

Proof. Firstly, let us remark that if the condition in the definition of deliberative acceptance of an argument a in an argumentation framework $AF = (A,D)$ is fulfilled only using maximal conflict-free sets of arguments, then it is fulfilled for arbitrary conflict-free sets. Indeed, if T is a conflict-free set attacking a, let T_0 be a maximal conflict-free set containing T. Clearly, $T \cap a^- \subseteq T_0 \cap a^-$, hence T_0 attacks a. Since T_0 is maximal, there is the conflict-free S such that $a \in S$ and $T_0 \cap S^- \subseteq S^+$. Let S_0 a maximal conflict-free set containing S. Clearly, $a \in S_0$ and $T_0 \cap S_0^- \subseteq S_0^+$. Hence $T \cap S^- \subseteq S_0^+$.

Since the underlying undirected graph G of AF is co-chordal, there are only $O(|A|)$ maximal conflict-free sets Tattacking a. The subgraph of G induced by $A - a^-$ is co-chordal, hence there are $O(|A|)$ maximal conflict-free sets containing a. Each such set S can be tested in polynomial time if $T \cap S^- \subseteq S^+$. It follows that in polynomial time we can decide if a is deliberatively accepted. □

5. Discussion

In this paper we introduced the *deliberative acceptability* of arguments which brings us closer to the intuitive dynamics of real world debates. We analyzed this type of acceptability and showed that depending on the combinatorial structure of the argumentation framework, the deliberative acceptance could agree to that based on extensions. We then investigated deliberative acceptability from a complexity view point and proposed graph theoretic restrictions under which this Π_2^{P}-complete problem belongs to P.

Using a formal treatment of its properties, we explained how the intuition behind the deliberative acceptability relates to the admissible extensions acceptability. In essence, both are based on the conflict-freeness and defense. We cannot use the general criteria for semantics evaluation as introduced in [3], since the different postulates defined there are for extension-based argumentation semantics and our semantics is not extension-based.

There are interesting future work lines opened by this approach. First of all, from Proposition 8 it follows that the deliberative acceptability is more concerned with credulous acceptance. However, we intend to introduce the skepticism in an usual way: an argument a is strongly deliberative acceptable if it is deliberative acceptable and no argument $b \in a^-$ is deliberative acceptable. Relating this kind of acceptability with skeptical extension based acceptability could be interesting in some applications.

Second, it would be interesting to identify generalizations of argumentation frameworks (GAF's) for which the new notion of acceptability have better modeling or computational qualities. More precisely, a GAF is a pair formed from an argumentation framework AF and a mechanism to specify efficiently a nonempty finite set of spanning subdigraphs of AF. This set of spanning subdigraphs may have only one element (as in Value-based Argumentation Frameworks (VAF's) [19]), or more elements (as in Weighted Argumentation Frameworks (WAF's) [20]). The goal will be to limit the number of conflict-free sets attacking an arbitrary argument.

Finally, we plan to investigate the argumentation frameworks having the underlying undirected graph the complement of an interval graph (the complement of an interval graph is a co-chordal graph – property used in Proposition 14). An interval graph could then be used to assign temporal intervals (see [21]) to the arguments, which can attack others arguments only if the associated intervals does not overlap. This could be an interesting introduction of temporal argumentation frameworks (TAF's) centered around deliberative acceptability.

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