Improved Robustness through Population Variance in Ant Colony Optimization

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Abstract. Ant Colony Optimization algorithms are population-based Stochastic Local Search algorithms that mimic the behavior of ants, simulating pheromone trails to search for solutions to combinatorial optimization problems. This paper introduces Population Variance, a novel approach to ACO algorithms that allows parameters to vary across the population over time, leading to solution construction differences that are not strictly stochastic. The increased exploration appears to help the search escape from local optima, significantly improving the robustness of the algorithm with respect to suboptimal parameter settings.

1 Introduction

Stochastic Local Search (SLS) [1] algorithms are an effective means to solve combinatorial optimization problems [2]. The Traveling Salesman Problem (TSP) is a well known combinatorial optimization problem where the goal is to construct the shortest possible tour visiting each city only once. As the number of cities increases, the combinatorial size prevents a complete search of the entire solution space. SLS algorithms employ diversification methods to find promising areas in the solution space and intensification methods to focus the search in these promising areas.

Ant Colony Optimization (ACO) [3] algorithms are population-based SLS algorithms where a colony of ants communicates indirectly through pheromone trails over a series of iterations. Each ant in the colony randomly constructs a solution to the problem using the pheromone trails and problem heuristics as aids. After each iteration, pheromone trail updates based on the best solutions found help narrow the search.

The performance of SLS algorithms depends on proper parameter selection. While parameter recommendations exist for these algorithms, the optimal parameters are often problem specific. This paper focuses on a new method to improve ACO algorithm robustness, the ability to perform well for suboptimal parameter selections [1].

Existing ACO algorithms employ a homogeneous colony of ants. The ants in these colonies use identical parameters throughout a run. The Population

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Variance (PV) approach introduced in this paper employs a heterogeneous colony of ants where key solution construction parameters vary across the colony during the run. This approach improves exploration of the solution space by the ACO algorithm, resulting in more robust performance with respect to suboptimal parameter settings.

The remainder of this paper uses the Max-Min Ant System [4] and Traveling Salesman Problem [2] to perform an empirical study of the Population Variance approach. Section 2 introduces the Population Variance approach. Section 3 studies the robustness of algorithm parameters when using Population Variance. Section 4 summarizes our findings and suggests future work.

2 Population Variance

Population Variance introduces the functions $\alpha_k(t)$ and $\beta_k(t)$ to the computation of proportional probabilities (1) used in solution construction. These functions allow us to change the values of α and β by ant k or iteration t, varying the relative contribution of the pheromones τ and the heuristics η.

$$p_{ij}^k(t) = \frac{[\tau_{ij\alpha}(t)]^{\alpha_k(t)} \cdot [\eta_{ij}]^{\beta_k(t)}}{\sum_{l \in N_i^k} [\tau_{il\alpha}(t)]^{\alpha_k(t)} \cdot [\eta_{il}]^{\beta_k(t)}}. \tag{1}$$

In this paper, we incorporate Population Variance into the Max-Min Ant System (MMAS) [4]. We previously proposed an improved lower limit for pheromone trails (2) in MMAS that avoids stagnation when computing the proportional probabilities and significantly improved results when $\alpha \neq 1$ [5]. We used this improved lower limit with both the MMAS and Population Variance algorithms studied in this paper.

$$\tau_{min} = \tau_{max} \cdot \left[\frac{1 - \sqrt[n]{p_{best}}}{avg \cdot \sqrt[n]{p_{best}}}\right]^{\frac{1}{\alpha}}. \tag{2}$$

The improved lower limit for pheromone trails in (2) implies that pheromone tables are now α specific, hence the use of $\tau_{ij\alpha}$ in (1). Pheromone scaling defined in (3) allows us to maintain a single pheromone table for $\alpha = 1$ and scale the pheromones proportionally for other values of α. This increases the computation of each proportional probability a small amount.

$$\tau_{ij\alpha}(t) = \tau_{min,\alpha} + (\tau_{ij1}(t) - \tau_{min,1}) \cdot \left[\frac{\tau_{max} - \tau_{min,\alpha}}{\tau_{max} - \tau_{min,1}}\right]. \tag{3}$$

There are many possible methods to select $\alpha_k(t)$ and $\beta_k(t)$. In this paper we use a simple diversification method to increase exploration, varying these values independently by iteration with a uniform distribution of d discrete values over a defined range for each. All ants share the same values for a given iteration, allowing construction of a single proportional probability table $p_{ij}^k(t)$. The $\alpha_k(t)$

function in (4) selects d discrete values between α_{min} and α_{max}. The $\beta_k(t)$ function in (5) selects d discrete values between $\beta_{lim}(t)$ and β_{max}.

$$\alpha_k(t) = \alpha_{min} + \left\lfloor \frac{\alpha_{max} - \alpha_{min}}{d-1} \right\rfloor \cdot \lfloor random(0,1) \cdot d \rfloor. \quad (4)$$

$$\beta_k(t) = \beta_{max} - \left\lfloor \frac{\beta_{max} - \beta_{lim}(t)}{d-1} \right\rfloor \cdot \lfloor random(0,1) \cdot d \rfloor. \quad (5)$$

Early prototypes showed an inverse relationship between the value of β and the quality of the initial solution. To intensify the search, we use $\beta_{lim}(t)$ to restrict the initial range of β values to produce better starting tours so that early pheromone updates guide us to productive areas of the solution space. The lower limit $\beta_{lim}(t)$ in (6) uses an exponential moving average from with decay rate σ and $\beta_{lim}(0) = \beta_{max}$.

$$\beta_{lim}(t+1) = (1-\sigma)\beta_{min} + \sigma\beta_{lim}(t). \quad (6)$$

These simple mechanisms for diversification and intensification will demonstrate the robustness of the Population Variance method. We plan to study more sophisticated mechanisms in the future.

3 Robustness

We modified ACOTSP [6] to incorporate the Population Variance equations and accept α_{min}, β_{min}, α_{max}, β_{max}, d, and σ parameters. A series of tests using TSPLIB [7] problems compared the performance of MMAS and PV across a range of parameters intended to provide optimal and suboptimal paremters combinations.

For MMAS, tests were run for all combinations of $\alpha = 1,2,3,4,5,6,7$ and $\beta = 1,2,3,4,5,6,7$. For PV, tests were run for $\alpha_{min} = 1$, $\beta_{min} = 1$, $\alpha_{max} = 2,3,4,5,6,7$, $\beta_{max} = 7$, $\sigma = 0.01$, and $d = 7$. For both algorithms, tests were run for all combinations of evaporation rates $\rho = 0.025, 0.05, 0.1, 0.3, 0.5$ and maximum pheromone selection probability $p_{best} = 0.00005, 0.0005, 0.005, 0.05, 0.5$ in addition to the α and β settings. All runs were limited to 10 tries of 2500 iterations. Local search was not employed so we could evaluate the effectiveness of the pheromone trail mechanism. All other parameters used their ACOTSP defaults.

PV exhibits more robust performance compared to MMAS across a range of parameter values for α_{max}, ρ, and p_{bests} as shown in Fig. 1 for problem $pcb442$ from TSPLIB. Similar results were obtained for other problems in TSPLIB. The MMAS tests for α_{max} include all tests with $\alpha \leq \alpha_{max}$ while the PV tests include a similar number of repeated tests for the given α_{max} value. The figure shows the same results for two ranges of percent deviation from optimal, $0-50$ and $0-10$. The range of PV results is much narrower and the PV medians are lower than the corresponding MMAS medians with a significance level less than 0.01 using the Mann-Whitney rank sum test. Tests varying other parameters

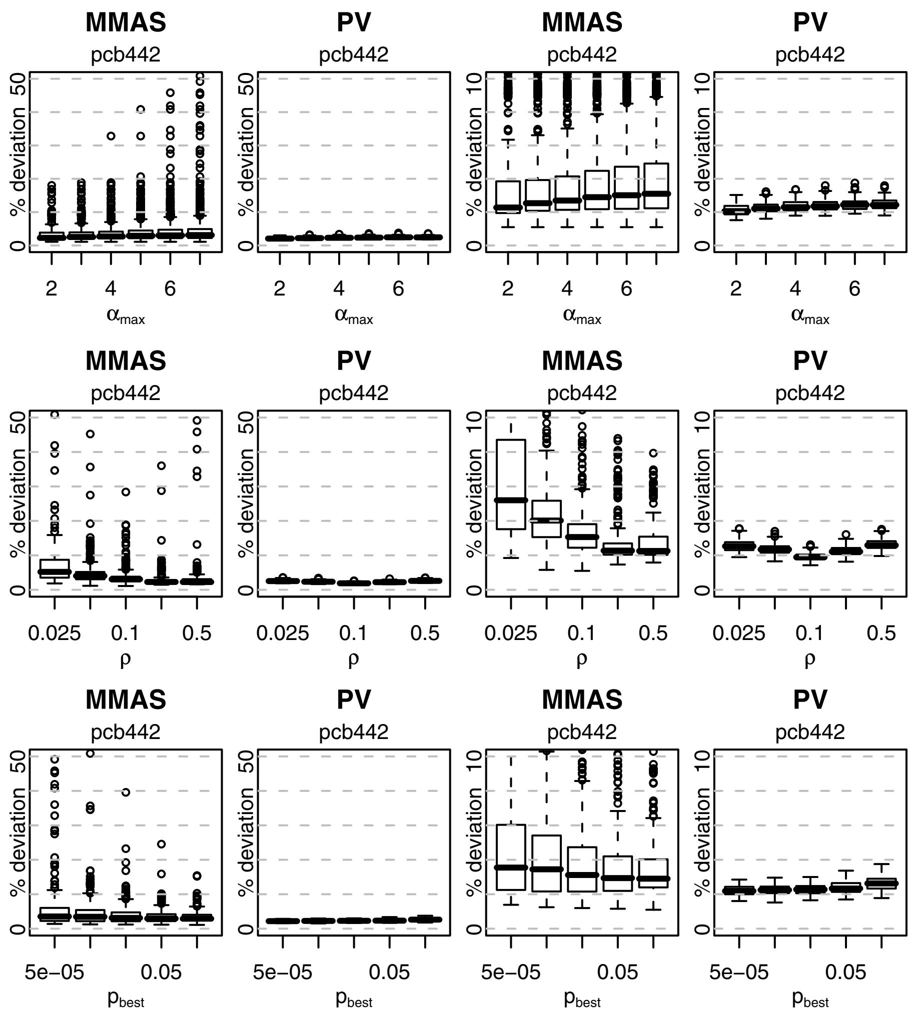

Fig. 1. Comparison of percent deviation from optimum of the Max-Min Ant System (MMAS) and Population Variance (PV) for ranges of α, ρ, and p_{best}

(pseudo random proportional selection, candidate list size, and number of ants) yielded similar improvements in robustness.

Some types of problems have no heuristics available to guide the search so we compared the performance of MMAS and PV using only pheromones in the random proportional selection, $\beta_{min} = \beta_{max} = 0$. The results in Fig. 2 shows four TSPLIB problems solved without the use of heuristics or local search, relying solely on the pheromone trails. These results suggest the PV methods for diversification and intensification are much more robust than MMAS.

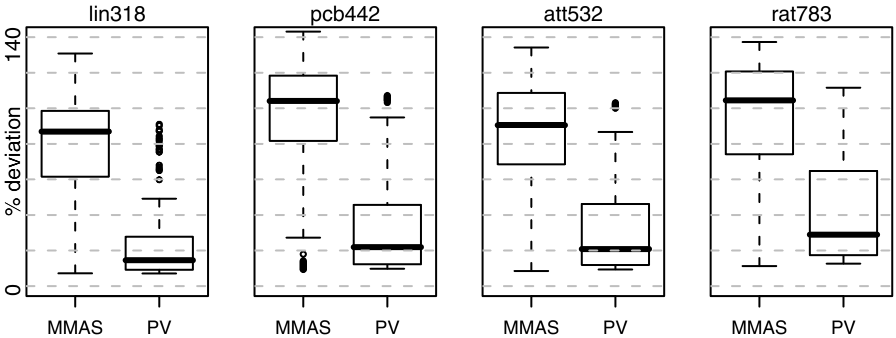

Fig. 2. Robustness for pheromone trails alone ($\beta = 0$)

4 Summary

This paper introduced a new method called Population Variance to increase robustness in ACO algorithms with respect to suboptimal parameter settings. This method varies the α and β parameters used during solution construction to improve exploration and escape local optima. The results of tests with problems from TSPLIB show significant improvements in robustness, particularly when heuristics are not available to aid the search.

Future work includes more sophisticated Population Variance functions $\alpha_k(t)$ and $\beta_k(t)$, interaction with local search, generalization to other ACO algorithms, and use with other types of combinatorial optimization problems.

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