Scale-Free Networks, Hyperbolic Geometry, and Efficient Algorithms

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Abstract

The node degrees of large real-world networks often follow a power-law distribution. Such scale-free networks can be social networks, internet topologies, the web graph, power grids, or many other networks from literally hundreds of domains. The talk will introduce several mathematical models of scale-free networks (e.g. preferential attachment graphs, Chung-Lu graphs, hyperbolic random graphs) and analyze some of their properties (e.g. diameter, average distance, clustering). We then present several algorithms and distributed processes on and for these network models (e.g. rumor spreading, load balancing, de-anonymization, embedding) and discuss a number of open problems. The talk assumes no prior knowledge about scale-free networks, distributed computing or hyperbolic geometry.

1998 ACM Subject Classification F.2.2 Nonnumerical Algorithms and Problems

Keywords and phrases power-law graphs, scale-free graphs, random graphs, distributed algorithms

Digital Object Identifier 10.4230/LIPIcs.MFCS.2016.4

Category Invited Talk

1 Short Review of Network Models

There are numerous models for large complex networks. The talk reviews some popular scale-free random graph models. The most cited network model are preferential attachment graphs by Barabási and Albert [2]. A bit more accessible for a formal analysis is the model of graphs with fixed expected degree sequences by Chung and Lu [10]. Both models follow a power-law degree distribution, but show only an extremely small clustering coefficient. Other models like the small-world model by Watts and Strogatz [24] generate local clustering, but do not converge to a power-law degree distribution.

There are a number of variations of the aforementioned models to generate graphs with power-law degree distribution *and* local clustering, but most are very artificial and therefore do not give an explanation *why* large networks typically show both properties. In the last couple of years it has been observed that complex scale-free network topologies with high clustering coefficients emerge naturally from hyperbolic metric spaces [23]. There seems to be a close relationship between hyperbolic geometry and complex networks. This can be explained by observing that the nodes of real-world networks can be often organized hierarchically, in an approximate tree-like fashion. Based on this and other observations, hyperbolic random graphs have been suggested in [23] and experimentally studied in [21]. Boguñá, Papadopoulos, and Krioukov [6] describe how hyperbolic mappings can be used to improve internet routing. Hyperbolic networks seem to combine all desired features of real networks in a natural model.

41st International Symposium on Mathematical Foundations of Computer Science (MFCS 2016).
Editors: Piotr Faliszewski, Anca Muscholl, and Rolf Niedermeier; Article No. 4; pp. 4:1–4:3
Leibniz International Proceedings in Informatics
LIPICS Schloss Dagstuhl – Leibniz-Zentrum für Informatik, Dagstuhl Publishing, Germany

2 Short Review of Algorithmic Results

The model of Chung and Lu [10] has been studied intensively. It has a giant connected component that contains a linear fraction of the nodes [10] and ultra-short average distances of $\mathcal{O}(\log \log n)$ [11, 12]. Algorithmically, these graphs have been examined in various contexts like information dissemination [14], bootstrap percolation [1], de-anonymization [7], and finding cliques [15]. Rumor spreading has also been studied on the preferential attachment model [13, 9]. Graphs can be generated from both models in linear time [22, 3].

For hyperbolic random graphs, much less is known so far. Besides the power-law degree distribution and high clustering [18, 21], the model generates larger cliques [16], polylogarithmic diameter [19, 17] and ultra-short average distances of order $\mathcal{O}(\log \log n)$ [8]. Hyperbolic random graphs can be generated in linear time [8]. In quasilinear runtime it is also possible to assign hyperbolic coordinates to large real-world graphs such that the hyperbolic metric approximates the graph distance [5]. Depending on the exponent β of the power-law degree distribution, the graphs have comparatively small separators and sublinear treewidth [4]. The model also allows fast bootstrap percolation [20].

References

1 H. Amini and N. Fountoulakis. What I tell you three times is true: Bootstrap percolation in small worlds. In *8th WINE*, pp. 462–474, 2012.

2 A.-L. Barabási and R. Albert. Emergence of scaling in random networks. *Science*, 286: 509–512, 1999.

3 V. Batagelj and U. Brandes. Efficient generation of large random networks. *Physical Review E*, 71:036113, 2005.

4 T. Bläsius, T. Friedrich, and A. Krohmer. Hyperbolic random graphs: Separators and treewidth. In *24th ESA*, 2016.

5 T. Bläsius, T. Friedrich, A. Krohmer, and S. Laue. Efficient embedding of scale-free graphs in the hyperbolic plane. In *24th ESA*, 2016.

6 M. Boguñá, F. Papadopoulos, and D. Krioukov. Sustaining the internet with hyperbolic mapping. *Nature Communications*, 1, 2010.

7 K. Bringmann, T. Friedrich, and A. Krohmer. De-anonymization of heterogeneous random graphs in quasilinear time. In *22nd ESA*, pp. 197–208, 2014.

8 K. Bringmann, R. Keusch, and J. Lengler. Geometric inhomogeneous random graphs. *arXiv preprint arXiv:1511.00576*, 2015.

9 F. Chierichetti, S. Lattanzi, and A. Panconesi. Rumor spreading in social networks. *Theoretical Computer Science*, 412:2602–2610, 2011.

10 F. Chung and L. Lu. Connected components in random graphs with given expected degree sequences. *Annals of Combinatorics*, 6:125–145, 2002.

11 F. Chung and L. Lu. The average distance in a random graph with given expected degrees. *Internet Mathematics*, 1:91–113, 2004.

12 S. Dereich, C. Mönch, and P. Mörters. Typical distances in ultrasmall random networks. *Advances in Applied Probability*, 44:583–601, 2012.

13 B. Doerr, M. Fouz, and T. Friedrich. Social networks spread rumors in sublogarithmic time. In *43rd STOC*, pp. 21–30, 2011.

14 N. Fountoulakis, K. Panagiotou, and T. Sauerwald. Ultra-fast rumor spreading in social networks. In *23rd SODA*, pp. 1642–1660, 2012.

15 T. Friedrich and A. Krohmer. Parameterized clique on scale-free networks. In *23rd ISAAC*, pp. 659–668, 2012.

16 T. Friedrich and A. Krohmer. Cliques in hyperbolic random graphs. In *34th INFOCOM*, pp. 1544–1552, 2015.

17 T. Friedrich and A. Krohmer. On the diameter of hyperbolic random graphs. In *42nd ICALP*, pp. 614–625, 2015.

18 L. Gugelmann, K. Panagiotou, and U. Peter. Random hyperbolic graphs: degree sequence and clustering. In *39th ICALP*, pp. 573–585, 2012.

19 M. Kiwi and D. Mitsche. A bound for the diameter of random hyperbolic graphs. In *12th ANALCO*, pp. 26–39, 2015.

20 C. Koch and J. Lengler. Bootstrap percolation on geometric inhomogeneous random graphs. In *43rd ICALP*, 2016.

21 D. Krioukov, F. Papadopoulos, M. Kitsak, A. Vahdat, and M. Boguñá. Hyperbolic geometry of complex networks. *Phys. Rev. E*, 82:036106, 2010.

22 J. C. Miller and A. Hagberg. Efficient generation of networks with given expected degrees. In *8th WAW*, pp. 115–126, 2011.

23 F. Papadopoulos, D. V. Krioukov, M. Boguñá, and A. Vahdat. Greedy forwarding in dynamic scale-free networks embedded in hyperbolic metric spaces. In *29th INFOCOM*, pp. 2973–2981, 2010.

24 D. J. Watts and S. H. Strogatz. Collective dynamics of 'small-world' networks. *Nature*, 393:440–442, 1998.