Int. J. Mach. Learn. & Cyber. (2013) 4:139–148
DOI 10.1007/s13042-012-0083-y

ORIGINAL ARTICLE

Weighted preferences in evolutionary multi-objective optimization

Tobias Friedrich · Trent Kroeger · Frank Neumann

Received: 16 January 2012 / Accepted: 13 February 2012 / Published online: 30 March 2012

Abstract Evolutionary algorithms have been widely used to tackle multi-objective optimization problems. Incorporating preference information into the search of evolutionary algorithms for multi-objective optimization is of great importance as it allows one to focus on interesting regions in the objective space. Zitzler et al. have shown how to use a weight distribution function on the objective space to incorporate preference information into hypervolume-based algorithms. We show that this weighted information can easily be used in other popular EMO algorithms as well. Our results for NSGA-II and SPEA2 show that this yields similar results to the hypervolume approach and requires less computational effort.

Keywords Evolutionary algorithms · Multi-objective optimization · User preferences

1 Introduction

Evolutionary algorithms are very powerful problem solvers especially when dealing with multi-objective optimization problems [5, 6]. Many successful methods have been developed in evolutionary multi-objective optimization (EMO) during recent years. Their basic goals are to compute a set of solutions that

(i) should be close to the true Pareto front,
(ii) should cover the complete Pareto front, and
(iii) should be uniformly distributed.

Popular evolutionary algorithms for multi-objective optimization are (among many others) NSGA-II [7] and SPEA2 [20], as well as hypervolume-based approaches such as SMS-EMOA [3] and MO-CMA-ES [13, 14].

Recently, there has been significant interest in strategies for introducing user preferences into EMO methods. The goal is to give specific regions of the objective space a higher priority and therefore relaxing goals (ii) and (iii) from above. Consequently, more solutions should be computed and maintained in the population for highly preferred regions of the objective space. For NSGA-II a reference point based approach has been proposed by Deb and Sundar [8]. They give the example that in the problem of maximizing throughput and minimizing latency, a decision maker may have a clue that throughput should be about 99.9%. Several authors [16, 1, 10] also used reference points to guide multi-objective particle swarm algorithms. Wickramasinghe and Li [17] use preferred areas. Auger et al. [2] present an approach of sampling the weighted hypervolume to incorporate user-defined preferences into the search for problems with many objectives. Thiele et al. [15] extended this such that at each iteration, a decision maker is asked to give preference information in terms of his reference point. Furthermore, [11] changed the crowding distance assignment in NSGA-II in order to achieve a non even spread of the points along the Pareto front.

We consider the case where there is a weight function on the objective space. This preference information can be used by the weighted hypervolume indicator instead of the

A conference version appeared in the Proceedings of the Australasian Conference on Artificial Intelligence 2011 [9].

T. Friedrich
Max-Planck-Institut für Informatik, Saarbrücken, Germany

T. Kroeger (✉) · F. Neumann
School of Computer Science, The University of Adelaide, Adelaide, Australia
e-mail: trent.kroeger@adelaide.edu.au

standard hypervolume indicator. Zitzler et al. [18] show that this weight integration is very well suited for incorporating user preferences. Furthermore, they compare their results to the ones obtained by NSGA-II and SPEA2 and show that these two algorithms do not perform well as they do not take into account the preference information.

In this paper, we present a neat and simple approach to use the preference information given by weightings on the objective space in classical algorithms such as NSGA-II and SPEA2. Up to now, this weight information has only been used by hypervolume-based approaches that have the drawback of needing a runtime exponential in the number of dimensions [4]. We present very simple approaches to incorporate the weight information on the objective space into a wide range of EMO algorithms. We exemplify this by using NSGA-II and SPEA2 and show that this leads to results similar to the ones of the weighted hypervolume indicator presented in [18]. Furthermore, our algorithms are as efficient as the original implementation of NSGA-II and SPEA2 and do not have to deal with the expensive computations that hypervolume-based algorithms have to face [4].

The outline is as follows. In Sect. 2, we introduce some basic concepts of multi-objective optimization and the weight functions used. Section 3 shows how to incorporate weight information into NSGA-II and SPEA2. We report our experimental results in Sect. 4 and finish with some concluding remarks.

2 Preliminaries

A multi-objective optimization problem is given by a vector-valued objective function

$$f = (f_1, \ldots, f_d)\colon S \to \mathbb{R}^d$$

on a search space S. W. l. o. g. we assume that each function f_i, $1 \leq i \leq d$, should be minimized. We first define a partial order on the objective space. An objective vector $x = (x_1, \ldots, x_d) \in \mathbb{R}^d$ weakly dominates an objective vector $y = (y_1, \ldots, y_d) \in \mathbb{R}^d$ ($x \preceq y$) if it is not worse in any objective, i.e., $x \preceq y :\Leftrightarrow x_i \leq y_i$ for $1 \leq i \leq d$.

Let $f(A)$ be the set of objective vectors of the search points in A, i.e., $f(A) = \{f(a)\colon a \in A\}$. Then, we denote by $\mathrm{Min}(f(A), \preceq)$ the set of minimal objective vectors in $f(A)$ with respect to the partial order $\preceq$ on $f(A)$. The goal in multi-objective optimization is to compute a set X^* with $f(X^*) = \mathrm{Min}(f(S), \preceq)$, where S is the considered search space. $f(X^*)$ is called the Pareto front of the given problem.

Often the size of the Pareto front is large, i.e., exponential with respect to the given input or even infinite in the case of continuous functions. In this case, it is not possible to compute the whole set of minimal elements of $f(S)$ efficiently and $f(X^*)$ should be a smaller subset of them.

So far, there has not been any preference between incomparable solutions. Having to cope with a large Pareto front, we have to decide between incomparable solutions. Basically, all successful evolutionary algorithms have certain diversity mechanisms to deal with this issue. Our goal is to investigate user preference in EMO. These user preferences give additional information for the search process and distinguish between sets of incomparable solutions.

We assume that we have access to a weight function $w\colon \mathbb{R}^d \mapsto \mathbb{R}$ which describes the preferences of the decision maker. In principle, w can be an arbitrary function that gives preferences to certain regions of the objective space. We will use the following weight distribution functions on the objective space which have been introduced and investigated by [18] in the context of hypervolume-based algorithms:

- Uniform weight: $w^{uni}(x) = 1$
- Sum of two exponential functions in the direction of the axes:

$$w^{ext}(x) = (e^{20 \cdot x_1} + e^{20 \cdot x_2})/(2 \cdot e^{20})$$

- Exponential function in the f_2-direction:

$$w^{asym}(x) = e^{20 \cdot x_2}/e^{20}$$

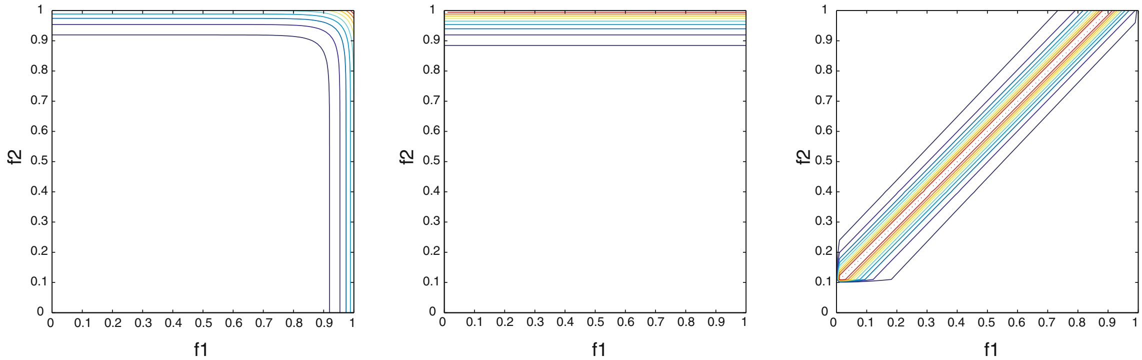
- Weighted depending on a reference point $ref = (a, b)$:

$$w^{ref}(x) = \begin{cases} c + \frac{(2-((2(x_1-a))^2+(2(x_2-b))^2))}{(0.001+(2(x_1-a)-2(x_2-b))^2)} \\ \quad \text{if } |x_1 - a| < 0.5 \wedge |x_2 - b| < 0.5 \\ c \text{ otherwise} \end{cases}$$

Note, that the uniform weight does not imply any preferences on the objective space. For NSGA-II and SPEA2 this will imply that we are just running the original versions of these algorithms. The weight distribution for the other three functions are illustrated in Fig. 1 for the objective space $[0, 1]^2$ and we will show in the following sections how to generalize NSGA-II and SPEA2 such that they can make use of this preference information.

3 Algorithms

In this section, we show how to integrate the user preferences given by weightings on the objective space into NSGA-II [7] and SPEA2 [20]. Both algorithms are based on the Pareto dominance relation and use diversity mechanisms to decide between incomparable solutions. We transfer the diversity mechanisms for incomparable solutions to the weighted case and adjust them such that they can make use of the weight information provided on the objective space.



(a) w^{ext} **(b)** w^{asym} **(c)** w^{ref} with $ref = (0.5, 0.6)$

Fig. 1 Contour plots of the weight distribution functions

3.1 Weighted NSGA-II

NSGA-II is a very popular evolutionary multi-objective algorithm. It is based on dominance ranking which ensures that non-dominated solutions are preferred over dominated ones. Furthermore, the algorithm has a diversity mechanism which distinguishes between incomparable solutions by a crowding distance measure. This measure prefers solutions of less crowded regions in the objective space.

Algorithm 1: Crowding assignment for weighted NSGA-II

- $\ell \leftarrow |X|$;
- **foreach** $x^j \in X$ **do**
  - $d(x^j) \leftarrow 0$;
- **for** $i \leftarrow 1$ **to** d **do**
  - Sort the solutions in P such that $f_i(x^1) \leqslant f_i(x^2) \leqslant \ldots \leqslant f_i(x^\ell)$ holds;
  - $d(x^1) \leftarrow d(x^\ell) \leftarrow \infty$;
  - **for** $j \leftarrow 2$ **to** $\ell - 1$ **do**
    - $d(x^j) \leftarrow d(x^j) + \frac{f_i(x^{j+1}) - f_i(x^{j-1})}{f_j^{\max} - f_j^{\min}}$;
- **foreach** $x^j \in P$ **do**
  - $d(x^j) \leftarrow w(x^j) \cdot d(x^j)$;

We will keep the dominance ranking as in the original algorithm. To incorporate the user preferences, we will change the crowding distance assignment according to the weight information on the objective space. Let X be a set of incomparable solutions then the assignment of a crowding distance, taking into account the weighting on the objective space, is given in Algorithm 1.

As the original NSGA-II, it iterates over all objectives. For each objective f_i, the solutions are sorted in increasing order and the distance of a solution is changed according to its neighboring points for that objective. Solutions that are maximal (or minimal) with respect to one objective obtain an infinite distance which gives strong preference to the extreme points of the Pareto front. Our weighted crowding distance assignment differs from the original crowding distance assignment by the last for-loop. In this loop the crowding distance of each solution is multiplied by the weight that its objective vector has according to the weight distribution on the objective space. Note, that this does not change the crowding distance assignment of the extreme points as they have already obtained an infinite distance. However, it changes the assignment and the preferences for the other points and gives preferences based on the weighting of the objective space. We use the weightening on the objective space to change the crowding assignment and incorporate the user preferences.

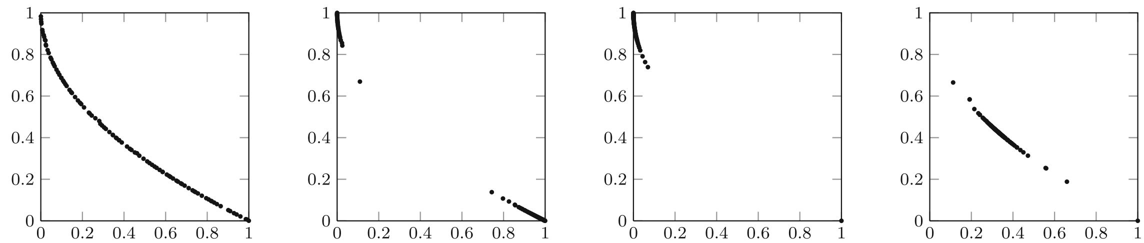
3.2 Weighted SPEA2

SPEA2 is another very popular approach. It sorts individuals of a population based on a fitness assignment strategy that incorporates both a coarse-grain evaluation of Pareto dominance that results in an integer raw fitness value and a fine-grained evaluation of density that allows the algorithm to distinguish between solutions with the same raw fitness. The sum of the density and raw fitness values yields the overall fitness of a solution, which is used within both the mating selection and environmental selection functions of the algorithm. The density $D(i)$ of a solution i is calculated as follows:

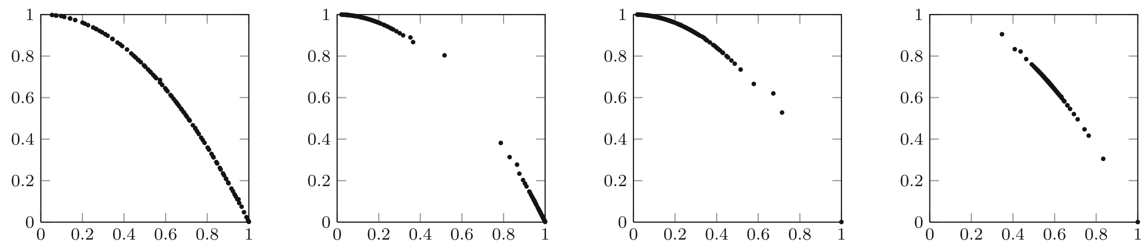
$$D(i) = \frac{1}{\sigma_i^k + 2}$$

where σ_i^k is the distance within the objective space from the solution i to its k th nearest neighbour in the population. Solutions in less crowded regions of the objective space will be assigned lower density values and will be preferred when compared to solutions in more crowded regions of the same Pareto front. In this way, SPEA2 maximises the diversity of solutions within the population. Note that the density function is constructed such that $D(i) \leq 0.5$ and as such, the density cannot affect Pareto dominance relationships between solutions, which have a raw fitness value of integer type.

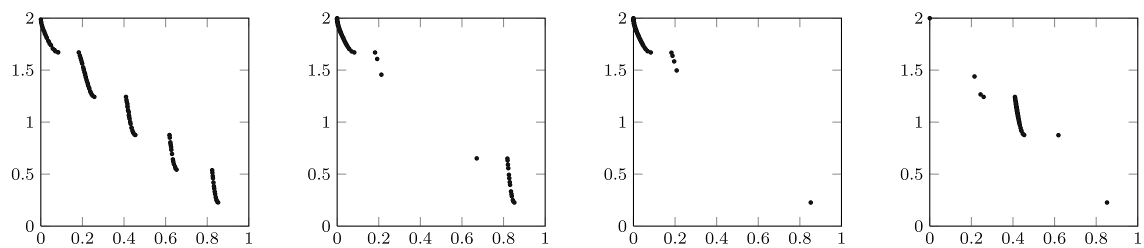
Our weighted density measure incorporates information from a weight distribution function as follows:



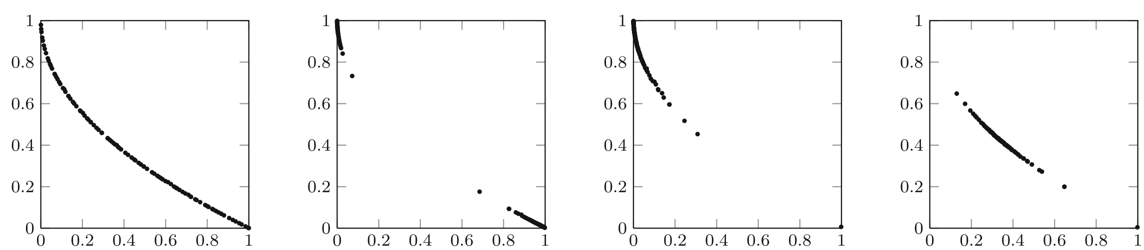
(a) NSGA-II on ZDT1 with weight functions w^{uni}, w^{ext}, w^{asym}, w^{ref}



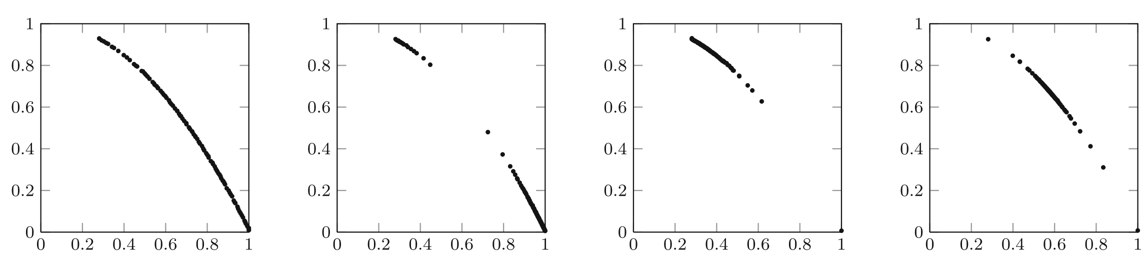
(b) NSGA-II on ZDT2 with weight functions w^{uni}, w^{ext}, w^{asym}, w^{ref}



(c) NSGA-II on ZDT3 with weight functions w^{uni}, w^{ext}, w^{asym}, w^{ref}

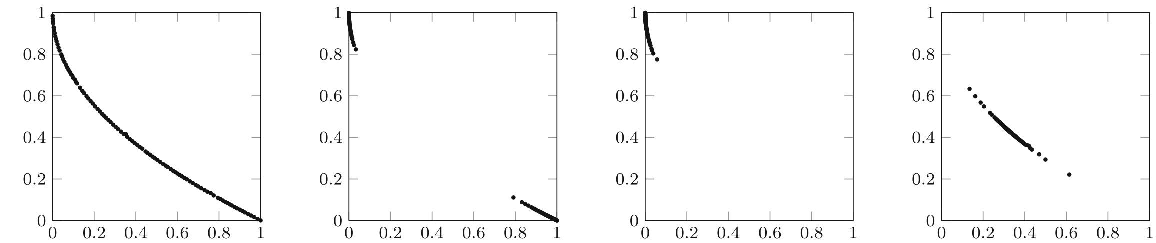


(d) NSGA-II on ZDT4 with weight functions w^{uni}, w^{ext}, w^{asym}, w^{ref}

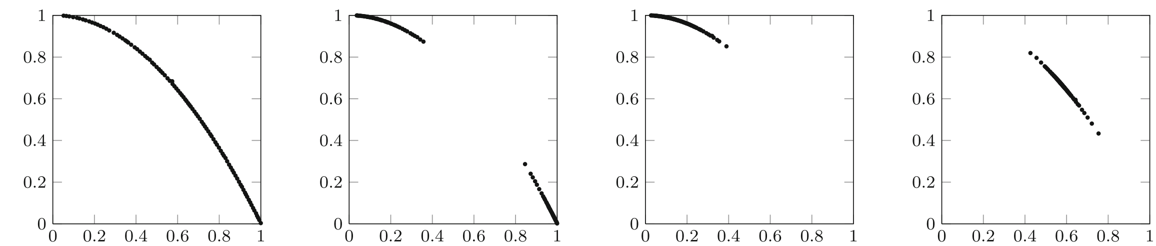


(e) NSGA-II on ZDT6 with weight functions w^{uni}, w^{ext}, w^{asym}, w^{ref}

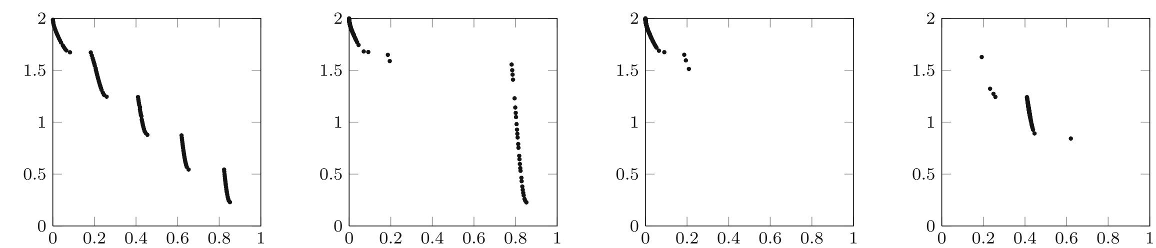
Fig. 2 Experimental results for NSGA-II on ZDT test functions



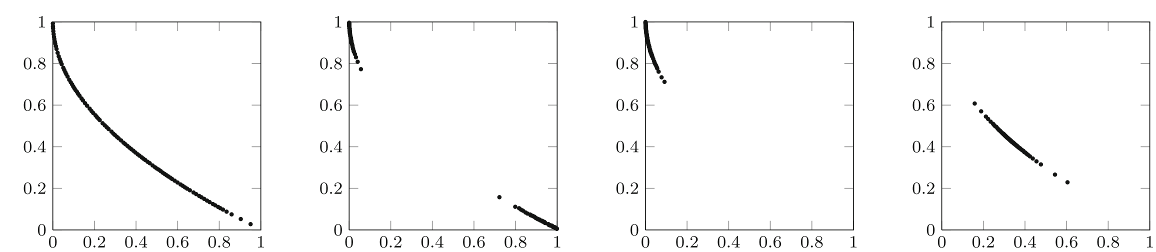
(a) SPEA2 on ZDT1 with weight functions w^{uni}, w^{ext}, w^{asym}, w^{ref}



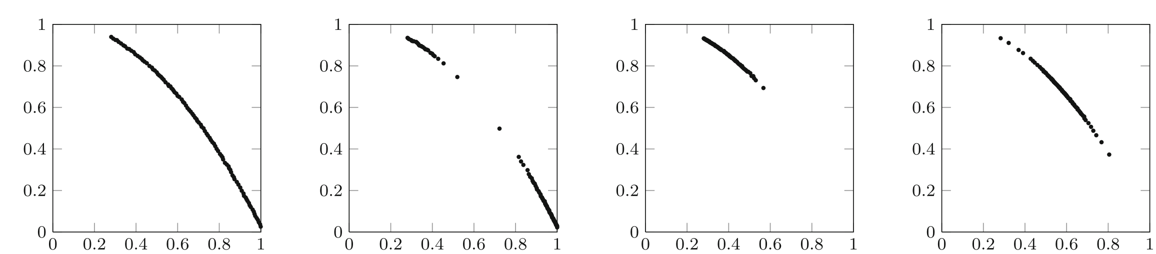
(b) SPEA2 on ZDT2 with weight functions w^{uni}, w^{ext}, w^{asym}, w^{ref}



(c) SPEA2 on ZDT3 with weight functions w^{uni}, w^{ext}, w^{asym}, w^{ref}

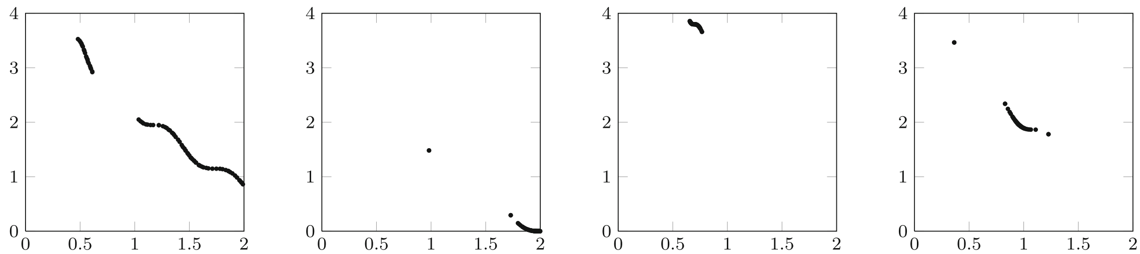


(d) SPEA2 on ZDT4 with weight functions w^{uni}, w^{ext}, w^{asym}, w^{ref}

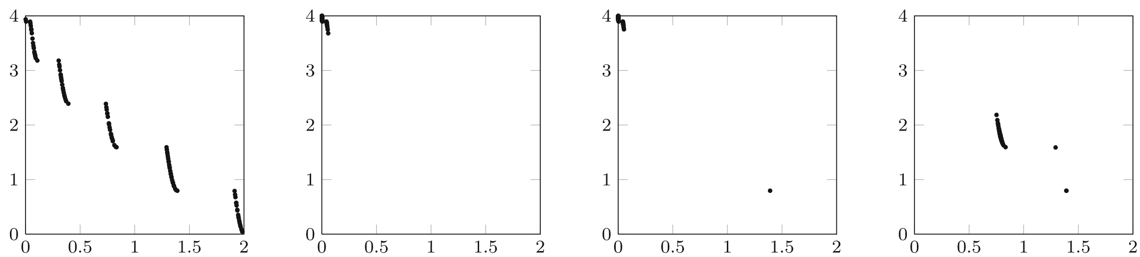


(e) SPEA2 on ZDT6 with weight functions w^{uni}, w^{ext}, w^{asym}, w^{ref}

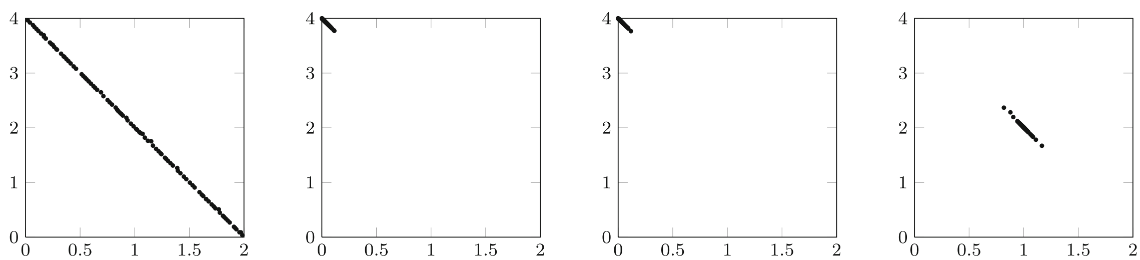
Fig. 3 Experimental results for SPEA2 on ZDT test functions



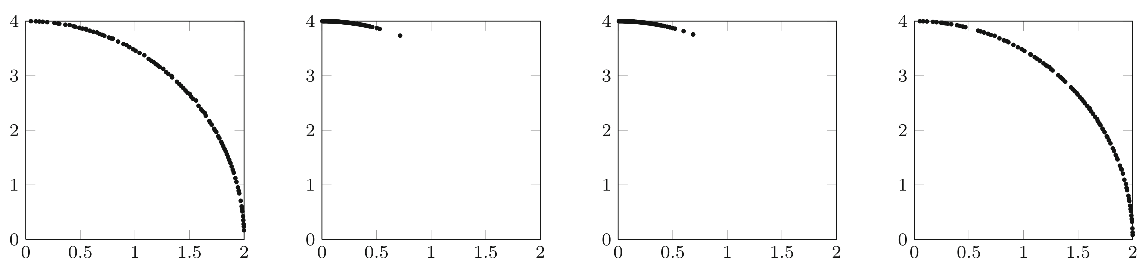
(a) NSGA-II on WFG1 with weight functions w^{uni}, w^{ext}, w^{asym}, w^{ref}



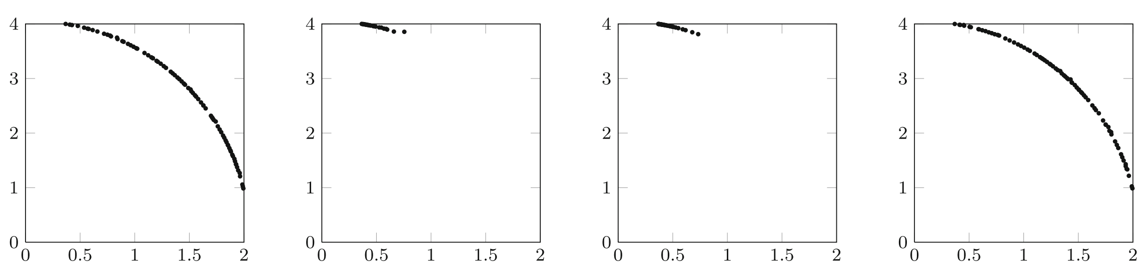
(b) NSGA-II on WFG2 with weight functions w^{uni}, w^{ext}, w^{asym}, w^{ref}



(c) NSGA-II on WFG3 with weight functions w^{uni}, w^{ext}, w^{asym}, w^{ref}

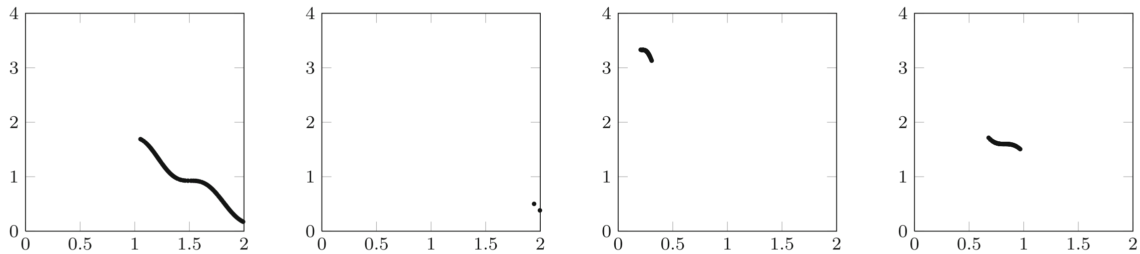


(d) NSGA-II on WFG4 with weight functions w^{uni}, w^{ext}, w^{asym}, w^{ref}

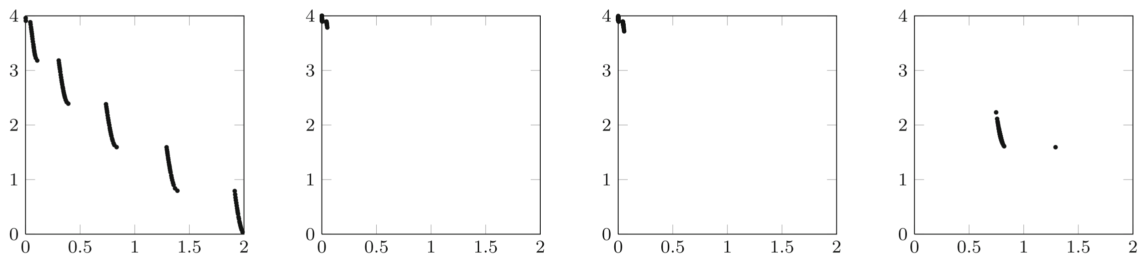


(e) NSGA-II on WFG5 with weight functions w^{uni}, w^{ext}, w^{asym}, w^{ref}

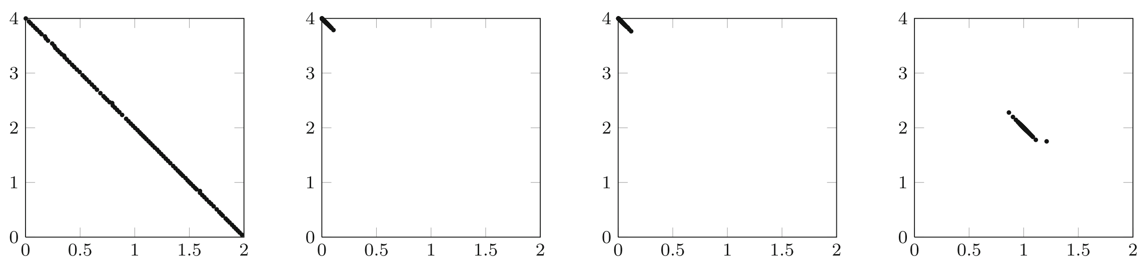
Fig. 4 Experimental results for NSGA-II on WFG test functions



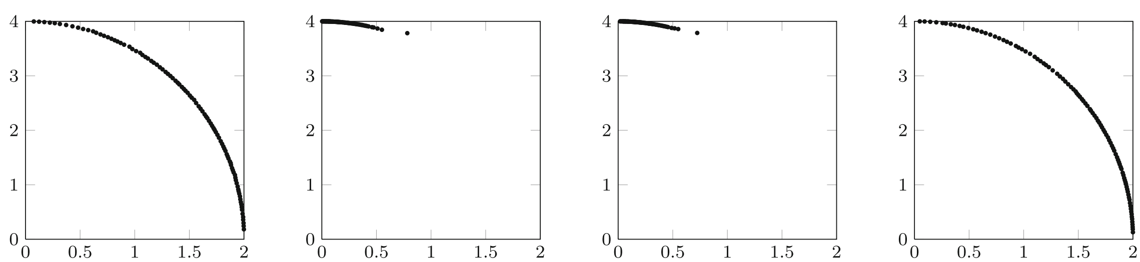
(a) SPEA2 on WFG1 with weight functions w^{uni}, w^{ext}, w^{asym}, w^{ref}



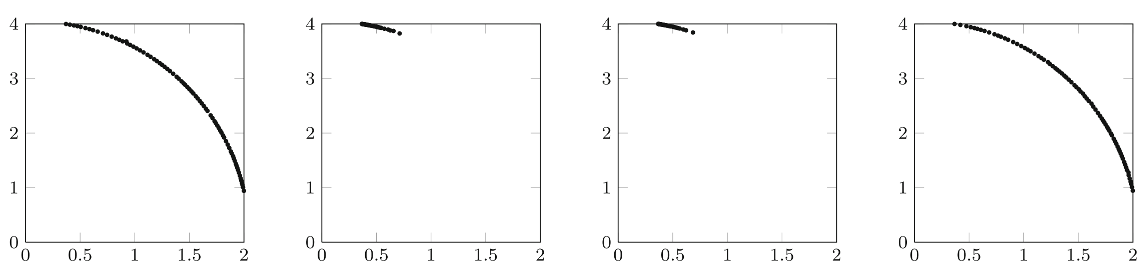
(b) SPEA2 on WFG2 with weight functions w^{uni}, w^{ext}, w^{asym}, w^{ref}



(c) SPEA2 on WFG3 with weight functions w^{uni}, w^{ext}, w^{asym}, w^{ref}



(d) SPEA2 on WFG4 with weight functions w^{uni}, w^{ext}, w^{asym}, w^{ref}



(e) SPEA2 on WFG5 with weight functions w^{uni}, w^{ext}, w^{asym}, w^{ref}

Fig. 5 Experimental results for SPEA2 on WFG test functions

$$D(i) = \frac{1}{(w_i \cdot \sigma_i^k) + 2}$$

where w_i is the value of the weight distribution function at the point in the objective space corresponding to solution i. Solutions corresponding to higher values of w_i will have a lower density $D(i)$ and will be favoured by the SPEA2 selection functions. Conversely, solutions corresponding to lower values of w_i will be less favoured by the SPEA2 selection functions. It is important to note that for any weight distribution function $w\colon \mathbb{R}^d \mapsto \mathbb{R}_+$, it still holds that $D(i) \leq 0.5$ and so, Pareto compliance is maintained.

For each iteration of the SPEA2 main loop, the environmental selection function involves copying non-dominated individuals from the archive and population at the previous iteration into a new archive. If the nondominated front fits exactly into the archive then the environmental selection step is completed. If there are not enough non-dominated individuals to fill the archive, the remaining places in the archive are filled with dominated individuals according to fitness. If there are too many non-dominated solutions to fit into the archive, then a truncation procedure is invoked to iteratively remove individuals until the non-dominated solutions fit within the archive. At each iteration of the truncation procedure, a solution i is chosen for removal that has the minimum distance to another individual. If there are several individuals with minimum distance then the second and, if necessary subsequent, smallest distances are considered to break the tie. Our weighted version of SPEA2 incorporates user preferences within the truncation procedure by multiplying the calculated distance from a solution i to its k th nearest neighbor by w_i to yield a weighted distance. The result of this modification is that solutions that are situated in highly weighted regions of the objective space will have a relatively high weighted distance and so will be less likely to be removed by the truncation procedure.

4 Experimental results

In this section, we report on our experimental results for the weight integration into NSGA-II and SPEA2. We use the same setting as [18] for the weighted hypervolume indicator and examine the classical benchmark functions ZDT1, ZDT3, and ZDT6 [19] and the weight distribution functions defined in Sect. 2. We also extend the experimentation to examine seven additional benchmark functions that include the ZDT functions ZDT2 and ZDT4, as well as test functions WFG1, WFG2, WFG3, WFG4 and WFG5 from [12].

We now examine how the three weight distribution functions defined above influence the search process of NSGA-II and SPEA2 for the five ZDT test problems. The functions ZDT1, ZDT2, ZDT3, ZDT4 and ZDT6, are optimized by NSGA-II runs with population size 100 for 25,000 generations and SPEA2 runs with population and archive sizes of 100 for 25,000 generations. For both algorithms, a crossover probability setting of 0.90 is used and mutation probability is set to 0.03. The test problems ZDT1, ZDT2, ZDT3 and ZDT6 were constructed with 30 decision variables and the function ZDT4 constructed with 10 decision variables. All test problems were formulated to be bi-objective.

Figures 2 and 3 show the computed Pareto front approximations for NSGA-II and SPEA2 after 25,000 generations for the five ZDT functions and the three weight distribution functions w^{ext}, w^{asym} and w^{ref}. The reference point for w^{ref} is chosen as $ref = (0.5,\ 0.6)$ for ZDT1, ZDT2, ZDT4 and ZDT6 and as $ref = (0.5,\ 1.2)$ for ZDT3 which is the same as in [18]. The difference in reference point position is due to the fact that the ZDT3 function has a larger range of values in the f_2-direction. The computed Pareto front approximation for a uniform weighting scheme w^{uni} is also shown to allow comparisons to the results of the original NSGA-II and SPEA2 variants.

Charts of the experiments show that for test functions ZDT1, ZDT2, ZDT4 and ZDT6, the weighted versions of NSGA-II and SPEA2 were highly successful in directing solutions towards regions of the objective space in accordance with all three of the weight distribution functions trialled. For the weighting scheme w^{ext}, both algorithms yielded a set of solutions that were concentrated near the boundary regions of the Pareto front. Similarly, the use of the weighting scheme w^{asym} resulted in a set of solutions concentrated near the boundary of the Pareto front in the f_2-direction. When the w^{ref} weight distribution function was used, both algorithms yielded results that were concentrated in a the region of the Pareto front that was closest to the specified reference point.

Similar behaviour was observed for the ZDT3 test function when the weighted NSGA-II and SPEA2 algorithms were executed using weighting schemes w^{asym} and w^{ref}. However, both algorithms produced less definitive results when using the ZDT3 test function and the w^{ext} weighting scheme. In particular, it appears that for the weighted SPEA2 algorithm, application of the w^{ext} weight distribution function led to a poor approximation of the true Pareto front in some regions.

We also examine how the three weight distribution functions influence the search process of NSGA-II and SPEA2 for the five WFG test problems. The functions WFG1, WFG2, WFG3, WFG4 and WFG5, are optimized by NSGA-II runs with population size 100 for 25,000 generations and SPEA2 runs with population and archive sizes of 100 for 25,000 generations. For both algorithms, a crossover probability setting of 0.90 is used and mutation probability is set to 0.03. Each WFG test problem was constructed for the bi-objective case with 2 position-related parameters and 4 distance related parameters.

Figures 4 and 5 show the computed Pareto front approximations for NSGA-II and SPEA2 after 25,000 generations for the five WFG functions and the three weight distribution functions w^{ext}, w^{asym} and w^{ref}. The reference point for w^{ref} is chosen as $ref = (1.0, 2.0)$ as the maximum values found along the pareto front for each function in the f_2-direction were approximately twice the maximum value in the f_1-direction. As for the ZDT functions, the computed Pareto front approximation for a uniform weighting scheme w^{uni} is also shown to allow comparisons to the results of the original NSGA-II and SPEA2 variants.

Charts of the WFG experiments show that for all test functions, the weighted versions of NSGA-II and SPEA2 were highly successful in directing solutions towards regions of the objective space in accordance with the w^{asym} and w^{ref} weight distribution functions. For the w^{ext} weighting functions, it was observed that the algorithms strongly preferenced solutions towards the f_2-direction of the objective space, rather than extremal points with respect to both objectives. It is likely that this is due to the fact that w^{ext} uses and exponential weighting scheme and the fact that the maximum values along the pareto front in the f_2-direction are approximately twice the maximum value in the f_1-direction. This combination of factors introduces a significant bias towards solutions in the f_2-direction of the objective space. Interestingly, this was not the case for the WFG1 test function, where both algorithms favoured solutions in the f_1-direction, when the w^{ext} weighting scheme was applied. A possible reason for this is the discontinuous nature of the WFG1 pareto front and the fact that its shape is more complex that the other WFG functions trialled.

Considering the performance on this range of test functions, the experimental results clearly demonstrate that the proposed weighted versions of NSGA-II and SPEA2 can be used successfully to guide solutions towards areas of the objective space according to an arbitrary weight distribution function. Importantly, these approaches are implemented in such a way that they can be used to incorporate user preferences without compromising Pareto compliance of the algorithms. Furthermore, weights are introduced to the diversity measures of each algorithm in such a way as to modify, but not completely destroy its diversity characteristics.

5 Conclusions

The integration of user preference into EMO methods is an important research topic as it allows the user of an EMO algorithm to focus on interesting regions of the objective space. Different models for the integration of user preference have been proposed. Incorporating user preferences by weight information on the objective space has been shown to work very well for hypervolume algorithms [18]. However, the hypervolume is highly expensive when dealing with higher dimensions.

We have presented a simple and very effective alternative way to use these user preferences in other state-of-the-art approaches such as NSGA-II and SPEA2. Our experimental results show that the weight integration into these algorithms performs very well, produces similar results as the weighted hypervolume indicator, and requires less computational effort than the hypervolume approach.

References

1. Allmendinger R, Li X, Branke J (2008) Reference point-based particle swarm optimization using a steady-state approach. In: Proc. simulated evolution and learning (SEAL '08). Lecture Notes in Computer Science, vol 5361, pp 200–209
2. Auger A, Bader J, Brockhoff D, Zitzler E (2009) Articulating user preferences in many-objective problems by sampling the weighted hypervolume. In: Proc. 11th annual conference on genetic and evolutionary computation, pp 555–562
3. Beume N, Naujoks B, Emmerich MTM (2007) SMS-EMOA: multiobjective selection based on dominated hypervolume. Eur J Oper Res 181:1653–1669
4. Bringmann K, Friedrich T (2010) Approximating the volume of unions and intersections of high-dimensional geometric objects. Comput Geom Theory Appl 43:601–610
5. Coello Coello CA, Van Veldhuizen DA, Lamont GB (2002) Evolutionary algorithms for solving multi-objective problems. Kluwer Academic Publishers, New York
6. Deb K (2001) Multi-objective optimization using evolutionary algorithms. Wiley, Chichester
7. Deb K, Agrawal S, Pratap A, Meyarivan T (2002) A fast and elitist multiobjective genetic algorithm: NSGA-II. IEEE Trans Evol Comput 6(2):182–197
8. Deb K, Sundar J (2006) Reference point based multi-objective optimization using evolutionary algorithms. In: Proc. 8th annual conference on genetic and evolutionary computation conference (GECCO '06), pp 635–642
9. Friedrich T, Kroeger T, Neumann F (2011) Weighted preferences in evolutionary multi-objective optimization. In: Wang D, Reynolds M (eds) Australasian conference on artificial intelligence. Lecture Notes in Computer Science, volume 7106, pp 291–300. Springer
10. Ho S-l, Yang S, Ni G (2008) Incorporating a priori preferences in a vector pso algorithm to find arbitrary fractions of the pareto front of multiobjective design problems. IEEE Trans Magn 44:1038–1041
11. Hu Q, Xu L, Goodman ED (2009) Non-even spread NSGA-II and its application to conflicting multi-objective compatible control. In: Proc. genetic and evolutionary computation conference summit (GEC '09), pp 223–230
12. Huband S, Barone L, While L, Hingston P (2005) A scalable multi-objective test problem toolkit. In: Evolutionary multi-criterion optimization, pp 280–295. Springer

13. Igel C, Hansen N, Roth S (2007) Covariance matrix adaptation for multi-objective optimization. Evol Comput 15:1–28
14. Suttorp T, Hansen N, Igel C (2009) Efficient covariance matrix update for variable metric evolution strategies. Mach Learn 75:167–197
15. Thiele L, Miettinen K, Korhonen PJ, Luque JM (2009) A preference-based evolutionary algorithm for multi-objective optimization. Evol Comput 17(3):411–436
16. Wickramasinghe UK, Li X (2008) Integrating user preferences with particle swarms for multi-objective optimization. In: Proc. 10th annual conference on genetic and evolutionary computation, pp 745–752
17. Wickramasinghe UK, Li X (2009) Using a distance metric to guide pso algorithms for many-objective optimization. In: Proc. 11th annual conference on genetic and evolutionary computation, pp 667–674
18. Zitzler E, Brockhoff D, Thiele L (2007) The hypervolume indicator revisited: on the design of Pareto-compliant indicators via weighted integration. In: Proc. fourth international conference on evolutionary multi-criterion optimization (EMO '07). LNCS, vol 4403, pp 862–876. Springer
19. Zitzler E, Deb K, Thiele L (2000) Comparison of multiobjective evolutionary algorithms: empirical results. Evol Comput 8(2):173–195
20. Zitzler E, Laumanns M, Thiele L (2002) SPEA2: Improving the strength Pareto evolutionary algorithm for multiobjective optimization. In: Proc. evolutionary methods for design, optimisation and control with application to industrial problems (EUROGEN 2001), pp 95–100