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WEAK TOTAL RESOLVABILITY IN GRAPHS

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Abstract

A vertex $v \in V(G)$ is said to *distinguish* two vertices $x, y \in V(G)$ of a graph G if the distance from v to x is different from the distance from v to y. A set $W \subseteq V(G)$ is a *total resolving set* for a graph G if for every pair of vertices $x, y \in V(G)$, there exists some vertex $w \in W - \{x, y\}$ which distinguishes x and y, while W is a *weak total resolving set* if for every $x \in V(G) - W$ and $y \in W$, there exists some $w \in W - \{y\}$ which distinguishes x and y. A weak total resolving set of minimum cardinality is called a *weak total metric basis* of G and its cardinality the *weak total metric dimension* of G. Our main contributions are the following ones: (a) Graphs with small and large weak total metric bases are characterised. (b) We explore the (tight) relation to independent 2-domination. (c) We introduce a new graph parameter, called *weak total adjacency dimension* and present results that are analogous to those presented for weak total dimension. (d) For trees, we derive a characterisation of the weak total (adjacency) metric dimension. Also, exact figures for our parameters are presented for (generalised) fans and wheels. (e) We show that for Cartesian product graphs, the weak total (adjacency) metric dimension is usually pretty small. (f) The weak total (adjacency) dimension is studied for lexicographic products of graphs.

Keywords: metric dimension, resolving set, weak total metric dimension, weak total resolving set, adjacency dimension, graph operations.

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