

# Hierarchical Image Computation with Dynamic Conjunction Scheduling

Christoph Meinel  
FB Informatik, University of Trier  
meinel@uni-trier.de

Christian Stangier  
FB Informatik, University of Trier  
stangier@uni-trier.de

## Abstract

*Image computation is the core operation for optimization and formal verification of sequential systems like controllers or protocols. State exploration techniques based on OBDDs use a partitioned representation of the transition relation to keep the OBDD-sizes manageable. This paper presents algorithms for building a hierarchically partitioned transition relation and conjunction scheduling based on this partitioning. The conjunction scheduling algorithm allows to dynamically reorder partitions and is targeted to improve the AndExist operation. Model checking experiments prove the effectiveness of the new algorithms.*

## 1. Introduction

The computation of the reachable states (RS) of a finite state machine (FSM) is an important task for synthesis, logic optimization and formal verification. The increasing complexity of sequential systems like controllers or protocols requires efficient RS computation methods. If the RS are computed by using Ordered Binary Decision Diagrams (OBDDs) [2], the system under consideration is represented in terms of a transition relation (TR). Since the monolithic representation of the circuit's TR usually leads to unmanageable large OBDD-sizes, the TR has to be partitioned [3, 6]. The quality of the partitioning is crucial for the efficiency of the RS computation. The computation of transitions will be unnecessarily time consuming, if the TR is divided into too many parts. On the other hand a number of partitions that is too small will lead to a blow-up of OBDD-size and hence, memory consumption.

The standard method is to sort the latches according to a benefit heuristic [7, 13] and then, apply a clustering algorithm. This clustering algorithm follows a greedy scheme [5] that is guided only by OBDD-size, i.e if the OBDD-size of a partition is exceeding a certain threshold a new partition has to be created.

Recently, new approaches for partitioning of the transition relation have been published: [9] presents a heuristic that minimizes *active lifetime* of the variables to gain a good conjunction schedule computed from a dependency matrix. Additionally the authors give a blocking strategy for the clustering. But, this method is restricted to forward image computation. In [11] and [12] heuristics are presented that focus on

grouping related variables to increase the quality of the partitioning, clustering takes place only within the given groups. The groups are determined from RTL descriptions resp. from a dependency matrix.

In this paper we extend the method of [11] to produce a real hierarchical partitioning of the transition relation. The hierarchical image computation is completed by an algorithm that performs the AndExist operation on the tree-like partitions that result from the hierarchical partitioning algorithm. The main impact comes from the heuristic that solves the problem of ordering the clusters for conjunction. It emerged that the AndExist algorithm works very well with a hierarchical partitioning and that this heuristic optimizes the performance of the AndExist. Additionally, the scheduling heuristic allows a true dynamic rescheduling of the partitions.

## 2. Preliminaries

Modern complex designs require a structured hierarchical description to be feasible. Often they are written in a hardware description language (HDL) at register transfer level (RTL). The term RTL is used for an HDL description style that utilizes a combination of *data flow* and *behavioral constructs*. Logic synthesis tools take the RTL HDL description to produce an optimized gate level netlist and high level synthesis tools at the behavioral level output RTL HDL descriptions. Verilog [15] and VHDL [8] are the most popular HDLs used for describing the functionality at RTL.

The design methodology in Verilog is a top down hierarchical modeling concept based on modules, which are the basic building block. Our experimental work is based on designs written in this language, but this approach can be easily extended to any hierarchical finite state machine representation as it is e.g. provided by state space decomposition algorithms (see. e.g. [10]).

### 2.1 Partitioned Transition Relations

The computation of the RS is a core task for optimization and verification of sequential systems. The essential part of OBDD-based traversal techniques is the transition relation TR:

$$\text{TR}(x, y, e) = \prod_i \delta_i(x_i, e) \equiv y_i,$$

which is the conjunction of the transition relations of all latches ( $\delta_i$  denotes the transition function of the  $i$ th latch).



One of the major benefits of this heuristic is that we are able to reduce the influence of the cluster-threshold resulting in a more robust partitioning. For comparison, when using the IWLS95 method we face a *butterfly effect* i.e. small changes in the cluster-size result in a large influence in the performance of the method (positive as well as negative).

The influence of the cluster-threshold has now been reduced to the clustering of a single (sub)module. But, we can reduce the influence even further: We introduced a *preclustering* step, where latches representing a multivalued register are clustered separately. Each multivalued register results in one or more clusters that are passed to the standard clustering routine described above. The impact of this preclustering was so evident that we increased the cluster-threshold for this step of the computation by a factor of two to allow more latches of a multivalued register to stay in one cluster. For comparison: increasing the standard cluster-threshold size of the IWLS method leads to a much poorer performance. See Figure 2 for a sketch of the clustering algorithm.

```

HierarchicalCluster(module,threshold){
  /* First, cluster the modules own latches */
  mv_relations =
    preclusterMVLatches(module→latches,threshold*2);
  latch_cluster =
    CreateClusters(mv_relations,threshold);

  append(cluster_array,latch_cluster);
  /* Then, cluster the children of the module */
  ForEachItem(module→children, child){
    child_cluster =
      HierarchicalCluster(child,threshold);
    append(cluster_array,child_cluster);
  }
  return cluster_array;
}

```

Figure 2: Algorithm in pseudocode for hierarchical partitioning.

- The benefits of the hierarchical partitioning heuristic are:
- We gain a less arbitrary and more structured partitioned transition relation.
  - The partitioning method is more robust, i.e. the cluster-threshold can be widely extended to increase performance for larger designs.
  - The heuristic performs excellently for structured design.
  - The heuristic is applicable to forward and backward image computation.

But one problem remains: The heuristic is not able to produce a schedule for conjunction of clusters during the And-Exist operation. Also, it seems unlikely that conjunction scheduling heuristics like [13, 9] improve the performance of this heuristics since their ordering strategies conflict with the grouping paradigm of this method.

## 4. Hierarchical Image Computation

In the following we will present algorithms to complete our framework for hierarchical image computation. The result of the algorithm *HierarchicalCluster* is a (linear) list of clusters that are not ordered (see. Figure 3a). This type of linearly arranged clusters is the same that we get from other partitioning algorithms (e.g. [13]). The image of a certain state set (represented by the OBDD  $S$ ) is obtained by consecutively applying AndExist ( $\odot$ ) to the OBDDs ( $T$ ) representing the transition relation. This algorithm is called “LinearAndSmooth”.

On the other hand, from *HierarchicalCluster* we obtain a basic ordering of clusters that are local to a certain (sub)module, and this is not adequately represented by a linear list.

Up to now no order for the processing of the submodules of a module has been computed. It is reasonable to think of an ordering for these submodules, since there is no way to detect an efficient schedule for processing from hierarchical description.

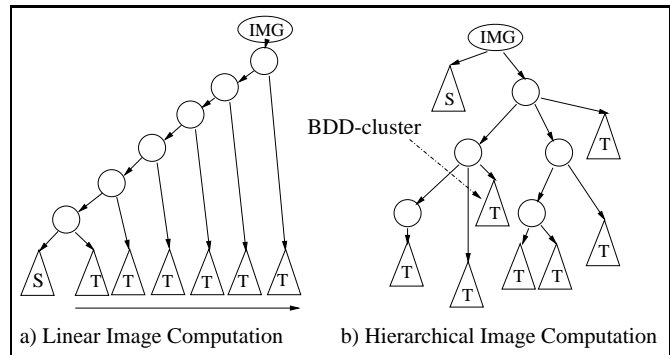


Figure 3: Linear and Hierarchical Image Computation.

### 4.1 General Algorithm

The algorithm outlined in Figure 4 describes the general way to compute an image hierarchically. To allow hierarchical image computation *HierarchicalCluster* has to be modified. The clusters are no longer put in a list, but stored in their according module (see. Figure 3b).

*HierarchicalAndSmooth* computes the image recursively in *preorder* style, i.e. first the module’s local clusters are conjuncted in the temporary product, then the computation continues with the submodules.

The preorder computation introduces the modules’ control variables first, resulting in an increase in the number of variables during the AndExist. On the other hand after finishing a submodule all variables that control only this module are quantified out, resulting in a decrease in OBDD-size.

```

HierarchicalAndSmooth(fromSet,module){
  product = fromSet;
  ForEachItem(img→cluster,cluster){
    smoothVars = ComputeSmoothVars(module,cluster);
    if (smoothVars)
      tmpProduct = bdd_and_smooth(product,cluster,
        smoothVars);
    else
      tmpProduct = bdd_and(product, cluster);
    product = tmpProduct;
  }
  childrenremaining = module→children;
  while(childrenremaining){
    child = ChooseBestSubmodule(childrenremaining);
    tmpProduct =
      HierarchicalAndSmooth(product,child);
    product = tmpProduct;
    remove_from(childrenremaining,child);
  }
  return product;
}

```

Figure 4: Algorithm in Pseudocode for Hierarchical Image Computation.

## 4.2 Dynamic Reordering of the Conjunction Schedule

The conjunction schedule for the image computation is determined in the *HierarchicalAndSmooth* by *ChooseBestSubmodule*, which can be computed statically or dynamically (the simplest solution would be the list order). Ordering heuristics like [13, 9] may be applied as well, but they are not useful for dynamic rescheduling as they only take structural information of the transition relation into account and will always result in the same schedule. Nevertheless, adjusting the conjunction schedule to changing state sets, OBDD-sizes, or variable orders might be very profitable.

We describe a strategy to improve the performance of the *AndExist* operation twofold: The *AndExist* operation generally profits from a hierarchical partitioning. And, we can use the hierarchy structure to improve the conjunction schedule dynamically.

The *AndExist* operation profits from a *compact* cube of smooth variables. The cube of smooth variables describes the set of variables that are quantified out during the *AndExist* operation. We call this cube compact, if the variables that appear in the cube are adjacent and not spread over the variable order of the OBDD. During a step of the *AndExist* recursion the following three cases are possible:

1. The current variable is contained in the smooth variable set: Then the recursion continues and the two resulting OBDDs are combined by an OR operation.

2. The current variable is not contained in the smooth variable set: The result is a new node labeled with the current index and whose successors are the results of the two recursions.
3. The cube has reached the sink node: The recursion reduces to an AND operation.

If the smooth variable cube is compact the third case appears earlier, improving the efficiency of the operation. And, if the clusters are separated, i.e. do not share many variables, then the third case may reduce to an identity function, because the cube and the cluster reach the sink node simultaneously.

This leads us to the following strategy for *ChooseBestSubmodule*:

1. Compute the maximum level (maxlevel) in the OBDD of a variable to be quantified out in all clusters and submodules of a given submodule.
2. Choose the submodules of the current module in increasing order of their maxlevels.

This strategy gives us a good schedule as we expect from the hierarchical partitioning that the clusters of the modules have highly separated variable sets resulting in compact cubes. Also, the schedule is changed dynamically as the variable order changes during the computation as a result of increasing state sets etc.

## 5. Experiments

We implemented our algorithms in the VIS-package [5] (version 1.3) using the underlying CUDD-package [14] (version 2.3.0). VIS is a popular verification and synthesis package in academic research. It inherits state of the art techniques for OBDD manipulation, image and reachable states computation as well as formal verification techniques. Together with the v12mv translator VIS provides a Verilog front-end.

### 5.1 Benchmarks

For our experiments we used Verilog designs from the Texas97 benchmark suite [1]. This publicly available benchmark suite contains real life designs from industry and academics including: MSI Cache Coherence Protocol, PCI Local BUS, PI BUS Protocol, MESI Cache Coherence Protocol, MPEG System Decoder, DLX and PowerPC 60x Bus Interface. The benchmark suite also contains properties given in CTL formulae for verification.

We chose those designs that represent RTL (i.e. including more than one module) rather than gate level descriptions. Only those designs were considered that could be read in and whose transition relation could be build respecting our system limitations. Table 1 shows 32 different benchmarks for which one or two sets of properties have been checked (resulting in 54 experiments). The runtime heavily depends on the chosen set of properties to be checked and is not proportional to the number of image computations. Therefore it is reasonable to check more than one set of properties. Some very small examples (CPU time < 10s) are not shown.

## 5.2 Experimental Setup

We left all parameters of VIS and CUDD unchanged. (Partition cluster size = 5000, partition method for MDDs = inout, OBDD variable reordering method = sifting, first reordering threshold = 4004 nodes) The model checking was preceded by a forced variable reordering. The CPU time was limited to 6 CPU hours and memory usage was limited to 200MB. All experiments were performed on Linux PentiumIII 500Mhz workstations.

## 5.3 Results

We compare our method (Hierarchy) to the standard method (IWLS95). For results on runtime and space requirements see Table 1. *lcmp* is the sum of forward and backward image computations performed during the analysis. *Parts* gives the number of partitions of the transition relation. The OBDD-size of the transition relation cluster and the peak number of live nodes is given by *TRn* resp. *Peakn*. The CPU time is measured in seconds and given as *Time*. The columns denoted with % describe the improvement in percent<sup>1</sup>.

At the bottom of Table 1 you can find the sum of all numbers of partitions, BDD-sizes and CPU-times. Also, the *total improvement* is given.

The experiments show significant improvements in time and space: The overall CPU time could be reduced to 1/4 of the original CPU time (11h instead of 45h). The hierarchical method outperforms the standard method in 51 of the 54 benchmarks. The decrease in computation time ranges up to 97%. The OBDD peak sizes could be lowered by 59% overall (20 million nodes instead of 50 million) Interestingly, the average OBDD size of a cluster reduced from 2464 nodes to 1761 nodes, although the threshold was doubled for multivalued registers. The overall number of clusters remains unchanged.

The effort for variable reordering during symbolic model checking is usually quite high, using the hierarchy method we were able to reduce, beyond all time improvements, the time fraction spent for variable reordering from 58% to 54% (overall).

## Conclusion

We have presented algorithms for partitioning of transition relations and conjunction scheduling. The partitioning algorithm uses hierarchical information to produce a tree-like clustered transition relation. We used RTL information given in Verilog, but any other algorithm that detects submodules of a FSM would work as well. The main impact is due to the algorithm that performs image computation based on this tree-like partitioning. This algorithm allows a dynamic rescheduling of the clusters, allowing to fine-tune the AndExist operation for a hierarchical partitioning. These al-

gorithms resulted in significant reductions in CPU-time and space.

The presented strategy for rescheduling only stands exemplarily for a wide variety of possible heuristics that may be implemented on the basis of the hierarchical partitioning, e.g. a history function that detects “expensive” AndExist operations and schedules them to a more suitable position.

## Acknowledgement

The authors would like to thank Jim Kukula for fruitful discussions and valuable ideas.

## References

- [1] A. Aziz et. al., *Texas-97 benchmarks*, <http://www-cad.EECS.Berkeley.EDU/Respep/Research/Vis/texas-97>.
- [2] R. E. Bryant, *Graph-Based Algorithms for Boolean Function Manipulation*, IEEE Transactions on Computers, C-35, 1986.
- [3] J. R. Burch, E. M. Clarke and D. E. Long, *Symbolic Model Checking with partitioned transition relations*, Proc. of Int. Conf. on VLSI, 1991.
- [4] J. R. Burch, E. M. Clarke, D. L. Dill, L. J. Hwang and K. L. McMillan, *Symbolic model checking: 10<sup>20</sup> states and beyond*, Proc. of Logic in Computer Science (LICS'90), 1990.
- [5] R. K. Brayton, G. D. Hachtel et.al., *VIS: A System for Verification and Synthesis*, Proc. of Computer Aided Verification (CAV'96), 1996.
- [6] O. Coudert, C. Berthet and J. C. Madre, *Verification of Synchronous Machines using Symbolic Execution*, Proc. of Workshop on Automatic Verification Methods for Finite State Machines, LNCS 407, Springer, 1989.
- [7] D. Geist and I. Beer, *Efficient Model Checking by Automated Ordering of Transition Relation Partitions*, Proc. of Computer Aided Verification (CAV'94), 1994.
- [8] R. D. M. Hunter and T. T. Johnson, *Introduction to VHDL*, Chapman & Hall, 1996.
- [9] I. Moon, G. D. Hachtel and F. Somenzi, *Border-Block Triangular Form and Conjunction Schedule in Image Computation*, Proc. of Formal Methods in CAD (FMCAD'00), LNCS 1954, 2000.
- [10] I. Moon, J. Jang, G. D. Hachtel, F. Somenzi, J. Yuan and C. Pixley, *Approximate Reachability Don't cares for CTL Model Checking*, Proc. of International Conference on CAD (ICCAD'98), 1998.
- [11] Ch. Meinel and C. Stangier, *Speeding Up Image Computation by using RTL Information*, Proc. of Formal Methods in CAD (FMCAD'00), LNCS 1954, 2000.
- [12] Ch. Meinel and C. Stangier, *A New Partitioning Scheme for Improvement of Image Computation*, Proc. of ASP Design Automation Conference (ASPDAC'01), 2001.
- [13] R. K. Ranjan, A. Aziz, R. K. Brayton, C. Pixley and B. Plessier, *Efficient BDD Algorithms for Synthesizing and Verifying Finite State Machines*, Proc. of Int. Workshop on Logic Synthesis (IWLS'95), 1995.
- [14] F. Somenzi, *CUDD: CU Decision Diagram Package*, <ftp://vlsi.colorado.edu/pub/>.
- [15] D. E. Thomas and P. Moorby, *The Verilog Hardware Description Language*, Kluwer, 1991.

<sup>1</sup>0 < improvement < 100; -100 < impairment < 0.

	IWLS95					Hierarchy							
	lcmp	Peakn	Parts	TRn	Time	Peakn	%	Parts	%	TRn	%	Time	%
ONE.pixley_cpu	113	24045	3	4456	12	29798	-19	4	-25	3220	27	13	-7
PClabnorm.PCI	304	176276	14	28613	253	116345	33	10	28	18061	36	152	39
PCInorm.PCI	206	81123	15	35124	56	69291	14	11	26	16993	51	47	16
TWO.PPCliveness	74	263756	8	13118	303	2083768	-87	10	-19	13881	-5	3436	-91
TWO.contention	37	97622	7	11865	47	215686	-54	10	-30	10508	11	150	-68
multi_main.multim	45	38694	5	14700	34	33423	13	6	-16	1578	89	18	47
p62_LS_LS_V01.ccp	64	166074	23	49952	200	124430	25	23	0	41246	17	84	58
p62_LS_LS_V01.p6l	99	452267	23	49952	831	158021	65	23	0	41246	17	173	79
p62_LS_LS_V02.ccp	54	146494	22	59487	105	117628	19	23	-4	42463	28	70	33
p62_LS_LS_V02.p6l	97	167454	22	59487	174	117628	29	23	-4	42463	28	79	54
p62_LS_LS_V01.cc	64	176540	23	49684	210	132128	25	22	4	41091	17	103	50
p62_LS_LS_V01.p6li	99	1617162	23	49684	3511	183674	88	22	4	41091	17	252	92
p62_LS_LS_V02.ccp	54	148560	23	62140	106	91257	38	23	0	37319	39	74	30
p62_LS_LS_V02.p6li	89	183811	23	62140	193	91257	50	23	0	37319	39	76	60
p62_LS_S_V01.ccp	64	176540	23	49684	210	132128	25	22	4	41091	17	103	50
p62_LS_S_V01.p6li	99	1614473	23	49684	3601	183674	88	22	4	41091	17	260	92
p62_LS_S_V02.ccp	54	148560	23	62140	106	91257	38	23	0	37319	39	74	30
p62_LS_S_V02.p6li	89	183811	23	62140	193	91257	50	23	0	37319	39	74	61
p62_LL_V01.ccp	52	164244	23	48961	189	117269	28	23	0	41165	15	78	58
p62_LL_V01.p6liv	89	477543	23	48961	934	189172	60	23	0	41165	15	159	82
p62_LL_V02.ccp	53	144504	23	48971	172	119677	17	23	0	41992	14	71	58
p62_LL_V02.p6liv	96	242452	23	48971	377	119677	50	23	0	41992	14	90	76
p62_LS_V01.ccp	75	168782	22	62479	121	123551	26	23	-4	42294	32	93	23
p62_LS_V01.p6liv	118	192410	22	62479	231	137714	28	23	-4	42294	32	166	28
p62_LS_V02.ccp	55	140767	22	57365	104	98699	29	22	0	40454	29	73	29
p62_LS_V02.p6liv	96	140767	22	57365	106	98699	29	22	0	40454	29	74	30
p62_ND_LS_V01.ccp	83	396642	24	63506	830	299289	24	24	0	46550	26	559	32
p62_ND_LS_V01.p6l	128	5583160	24	63506	21039	1747044	68	24	0	46550	26	4544	78
p62_ND_LS_V02.ccp	103	191386	22	63321	331	156783	18	23	-4	44262	30	175	47
p62_ND_LS_V02.p6l	192	1564426	22	63321	3611	445241	71	23	-4	44262	30	669	81
p62_ND_LL_V01.ccp	75	356794	25	65964	781	380121	-6	24	4	45076	31	577	26
p62_ND_LL_V02.ccp	161	5614430	23	60383	timeout	1352990	75	23	0	47981	20	3050	85
p62_ND_LL_V02.p6li	200	5573568	23	60383	timeout	3009524	46	23	0	47981	20	4965	77
p62_ND_S_V02.ccp	84	150630	23	46744	188	133586	11	24	-4	41048	12	142	24
p62_ND_S_V02.p6li	177	645917	23	46744	1231	164486	74	24	-4	41048	12	200	83
p62_S_S_V01.ccp	43	147063	23	62209	101	97360	33	23	0	39901	35	62	38
p62_S_S_V01.p6liv	80	153012	23	62209	106	97360	36	23	0	39901	35	67	36
p62_S_S_V02.ccp	37	129492	23	54800	94	90710	29	23	0	39819	27	60	36
p62_S_S_V02.p6liv	74	129492	23	54800	95	90710	29	23	0	39819	27	61	35
p62_V_LS_V01.ccp	108	283494	24	58415	587	210147	25	23	4	45928	21	362	38
p62_V_LS_V01.p6li	153	4483034	24	58415	timeout	2126524	52	23	4	45928	21	6236	71
p62_V_LS_V02.ccp	90	165200	23	52073	221	128016	22	22	4	41985	19	148	33
p62_V_LS_V02.p6li	178	1059895	23	52073	1864	224255	78	22	4	41985	19	286	84
p62_V_S_V01.ccp	82	213245	23	61795	245	142930	32	23	0	43513	29	173	29
p62_V_S_V01.p6liv	127	964988	23	61795	2168	442596	54	23	0	43513	29	890	58
p62_V_S_V02.ccp	84	163439	22	54807	189	126542	22	23	-4	41969	23	120	36
p62_V_S_V02.p6liv	177	351553	22	54807	475	228621	34	23	-4	41969	23	241	49
packet.packet	65326	53790	3	9704	5122	68473	-21	4	-25	4742	51	5068	1
single_main	108	14936	2	6352	13	9360	37	4	-50	884	86	8	38
single_main.1	52	14936	2	6352	13	9360	37	4	-50	884	86	7	46
three_processor.p	244	4621235	9	19750	timeout	3062696	33	7	22	4838	75	2959	86
three_processor_bin.	140	8779857	7	20387	timeout	560970	93	7	0	5140	74	522	97
two_processor.pro	264	903917	4	12311	676	88215	90	5	-19	1810	85	72	89
two_processor_bin	141	252974	4	11610	150	64924	74	5	-19	2623	77	42	72
Sum:	70748	50497236	1022	2518138	160514	20625941		1027		1809018		38307	
Total Improvement:						59%		0%		28%		76%	

Table 1: Comparison of IWLS95 Method and Hierarchy Heuristic