Near-Optimal Deterministic Single-Source Distance Sensitivity Oracles

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UNIVERSITÀ DEGLI STUDI DI SASSARI

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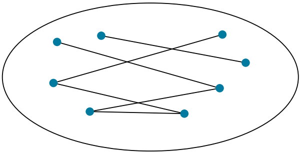
Fault-Tolerant Data Structures

a.k.a. sensitivity data structures, algorithm for emergency planning, or failure-prone graphs



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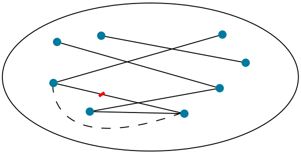

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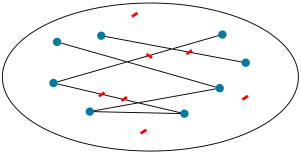

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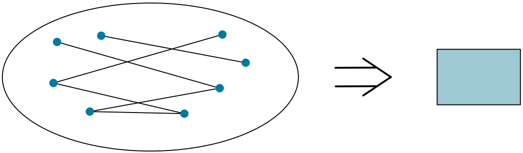

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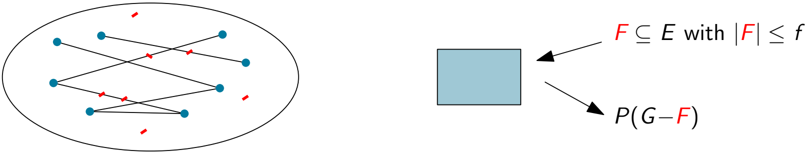

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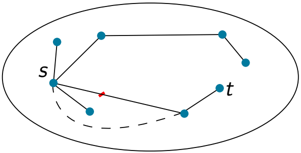

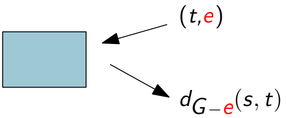

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This talk: $P(G) = d_G(s, \cdot)$ - distances from source s, for $f = 1$ failure.

Single-source distance sensitivity oracle (single-source DSO).

Single-Source Replacement Paths Problem

Undirected graphs, n vertices, m edges, $m \geq n$, space in $O(\log n)$-bits machine words, $\widetilde{O}(.)$ hides polylog(n).
DSO - distance sensitivity oracle, SSRP - single-source replacement paths.

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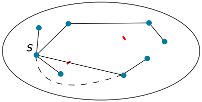
$\Rightarrow$ Output size $O(n^2)$.



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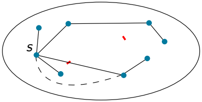

Undirected graphs:

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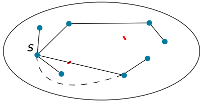

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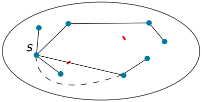

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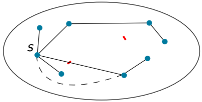

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Tabulation: single-source DSOs with $O(n^2)$ space and $O(1)$ query time.

Subquadratic Space Single-Source DSOs

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- Bilò et al. [STACS 2018]: $\widetilde{O}(n^{3/2})$ space single-source DSO.
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Our Result

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Upper Bounds

There are deterministic single-source DSOs with $\widetilde{O}(1)$ query time and

(i) combinatorial $\widetilde{O}(m\sqrt{n} + n^2)$ preprocessing time
for unweighted graphs, taking $O(n^{3/2})$ space;

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Space Lower Bound

Any single-source DSO must take $\Omega(M^{1/2}n^{3/2})$ *bits* of space on n-vertex graphs with integer edge weights in $[1, M]$, unconditionally.

Reducing Single-Source DSOs to SSRP

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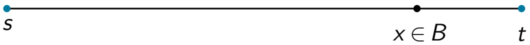

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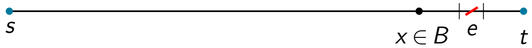

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HPI

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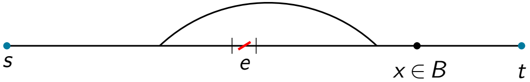

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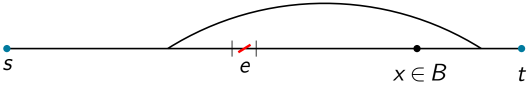

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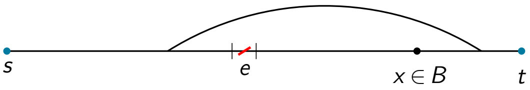

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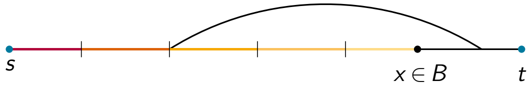

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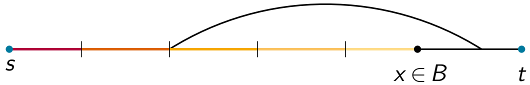

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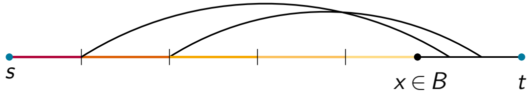

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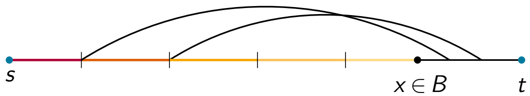

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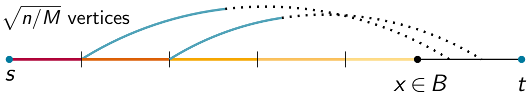

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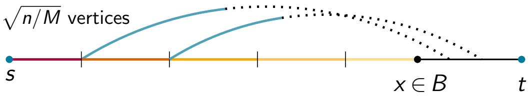

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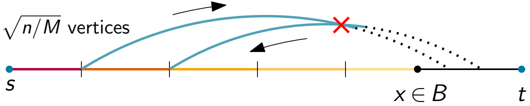

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- Identify long representatives by their *stub* - first $\sqrt{n/M}$ vertices on detour.
- Stubs cannot intersect! (So there are only $\sqrt{Mn}$ stubs.)
  - Otherwise segment has replacement path of length $< d_G(s,t) + 2\sqrt{Mn}$.

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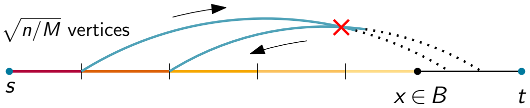

Claim: $O(\sqrt{Mn})$ segments of same replacement distance.

- *Respresentative* replacement paths with distinct lengths $\geq d_G(s,t)$.
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- Identify long representatives by their *stub* - first $\sqrt{n/M}$ vertices on detour.
- Stubs cannot intersect! (So there are only $\sqrt{Mn}$ stubs.)
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At most $3\sqrt{Mn}$ representatives/segments/distances in far case II.

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Compressing the Far Case II

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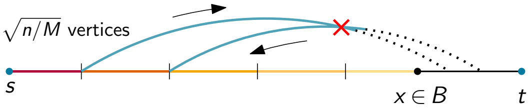

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Also holds for vertex failures, but crucially depends on G being undirected.

Derandomization & Subquadratic Preprocessing

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Subquadratic Preprocessing

For undirected graphs with integer edge weights in $[1, M]$ having $m = O(n^{(5/4)-\varepsilon}/M^{7/4})$ edges, there is a *randomized* single-source DSO with $\widetilde{O}(n^{2-\varepsilon/2})$ preprocessing time, $\widetilde{O}(1)$ query time, and $O(M^{1/2}n^{3/2})$ space.

Conclusion

1. Improved deterministic single-source DSOs for undirected graphs with near-optimal preprocessing time, query time, and space.

2. First subquadratic preprocessing algorithm for sparse graphs.

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Thank you.