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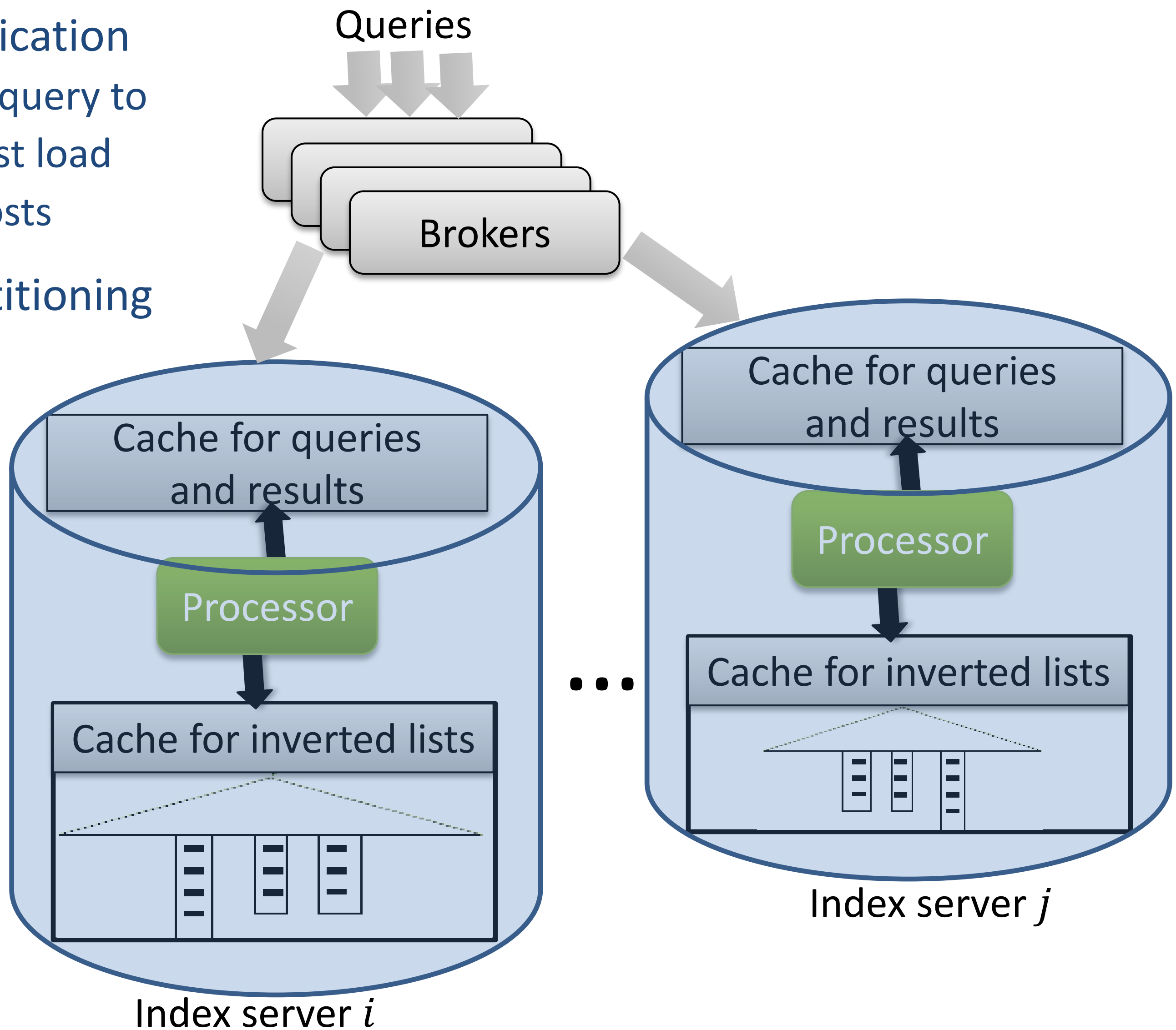
FROM QUERIES TO TOP-K RESULTS

Outline

- Intro
- Basics of probability and information theory
- Retrieval models
- Retrieval evaluation
- Link analysis
- **From queries to top-k results**
  - Query processing
  - Index construction
  - **Top-k search**
- Social search

Distributed index maintenance (overview)

- Inverted index replication
  - Broker forwards query to server with lowest load
    → high resource costs
- Inverted Index partitioning
  - By documents
  - By terms
  (Work of brokers depends on partitioning strategy)
- Variations of LRU strategy for dropping data from cache



Index partitioning strategies

- **Partitioning by documents** ("horizontal partitioning": inverted lists are partitioned)
  - Vocabulary is replicated on all servers (i.e., nodes)
  - Inverted list entries are hashed onto nodes by document IDs
  - Query is forwarded to each node and results are merged
    - → easy to maintain, scalable, load-balanced,
    - → resource-consuming

- **Partitioning by terms** ("vertical partitioning": vocabulary is partitioned)
  - Vocabulary is (partitioned and) distributed across multiple nodes
  - Inverted lists are mapped onto nodes responsible for the corresponding terms
  - Query is send to nodes with relevant terms

    What are the consequences for maintenance, scalability, load-balancing, resource-consumption?

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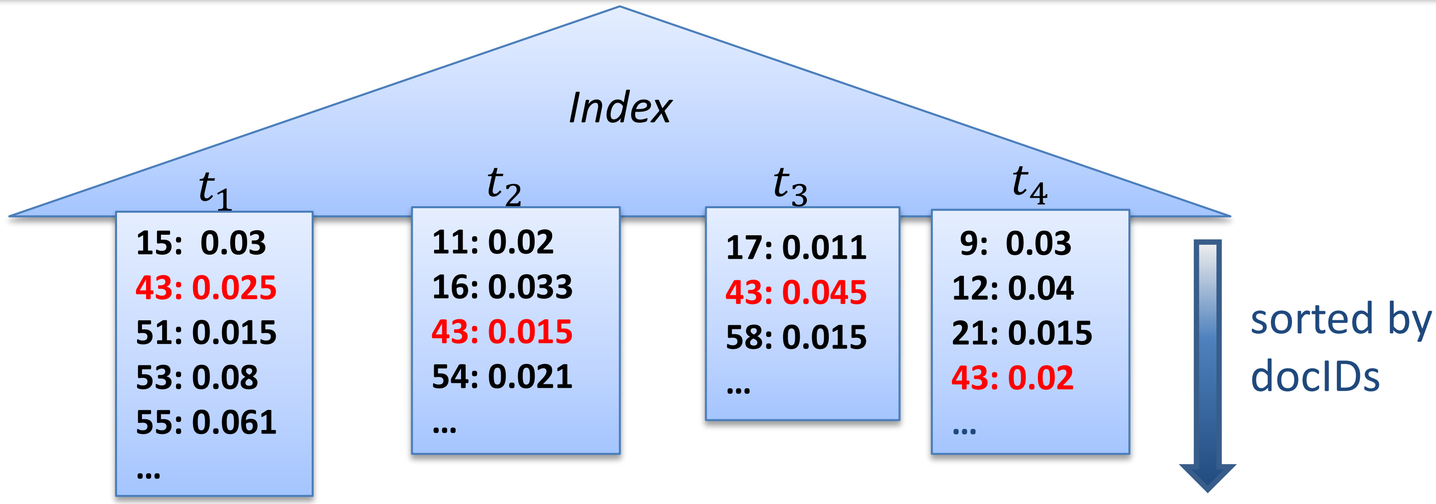
Computing top-k results (1)

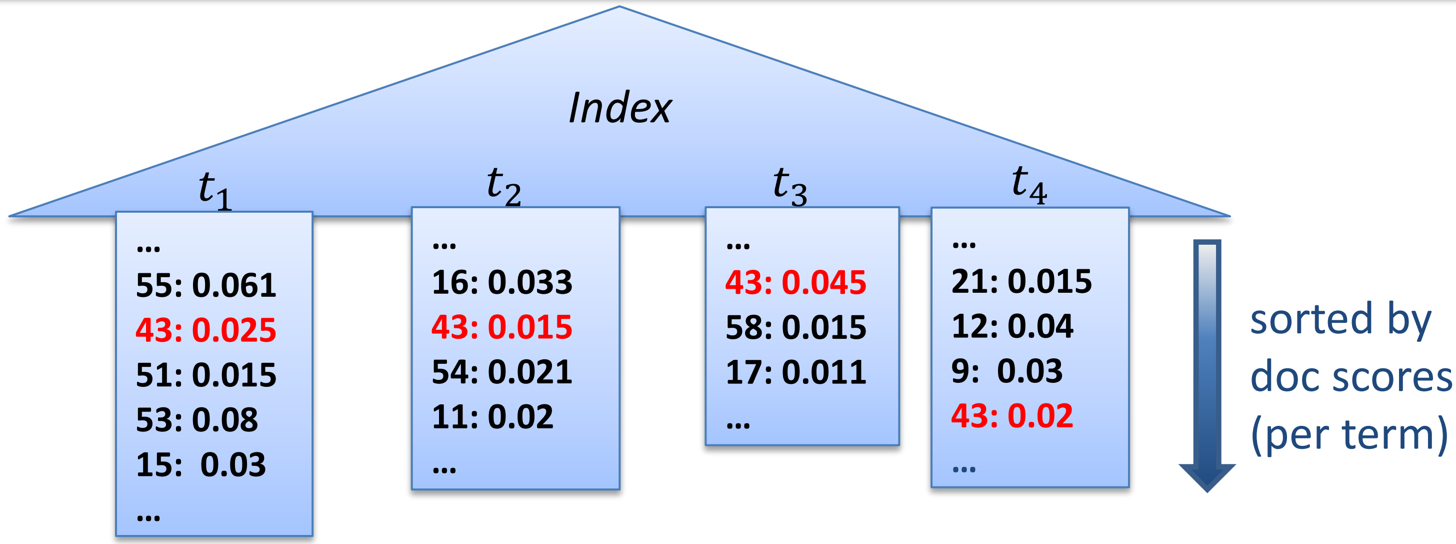


- **Top-k join-and-sort** for **Boolean queries** on virtual relations of the form $Index\ (term, docID, Sc)$
  - **Input:** query $q = t_1\ t_2\ \ldots t_l$
  - **Required:** top-k docs $d_1, d_2, \ldots, d_k$ ranked by some match score:
  $\forall\, i, 1 < i \leq k, \forall j > k$: $Sc(d_i, q) \leq Sc(d_{i-1}, q) \wedge Sc(d_i, q) \geq Sc(d_j, q)$

top-k{ $\sigma_{[term=t_1]}(Index) \bowtie_{docID}$
$\sigma_{[term=t_2]}(Index) \bowtie_{docID}$
… $\bowtie_{docID}$
$\sigma_{[term=t_l]}(Index)$ order by Sc desc}

Most efficient when inverted list entries are sorted by docID!

Computing top-k results (2)



- **Top-k join** with **score aggregation** on virtual relations of the form $D_1(docID, score_{t_1}), \ldots, D_l(docID, score_{t_l})$
  - **Input:** query $q = t_1\ t_2\ \ldots t_l$
  - **Required:** top-k docs $d_1, d_2, \ldots, d_k$ ranked by some match score:
  $\forall\, i, 1 < i \leq k, \forall j > k\colon\ Sc(d_i, q) \leq Sc(d_{i-1}, q) \wedge Sc(d_i, q) \geq Sc(d_j, q)$

Select $docID$, $Sc(D_1.score_{t_1}, \ldots, D_l.score_{t_l})$ As $Score$
From Outer Join $D_1, \ldots, D_l$
Order By $Score$ Limit k

If Sc is monotone, simple and principled algorithms exist.

Top-k processing of score-ordered inverted lists

- Assumptions
  - List entries sorted by per-term doc scores
  - Scoring function $Sc(a_1, \ldots, a_l)$ is monotone
    $(a_1 \geq b_1) \wedge \cdots \wedge (a_l \geq b_l) \Rightarrow Sc(a_1, \ldots, a_l) \geq Sc(b_1, \ldots, b_l)$

- General heuristics
  1. Scan lists in sequentially and in Round-Robin fashion (disregard lists with term-idf score below some threshold or prioritize short lists)
  2. If possible (i.e., when the whole lists are in main memory) perform random access to entries with same docID in other lists
  3. Compute scores for docs incrementally, as more **dimensions** (i.e., per-term scores) are observed
  4. Stop when top-k docs are found (heuristically: until all dimensions are seen for k'>k docs)

Threshold algorithm (Fagin et al. 2001*)

- All inverted lists $L_1, \ldots, L_l$ are sorted by tf
- Random access to each list is possible

Do sorted access in parallel to all lists

Let cdim_i be the last position visited in sorted access in each L_i

Define threshold $T = Sc(\mathrm{cdim}_1.score, \ldots, \mathrm{cdim}_l.score)$

If new doc d is seen in one of the lists

Find all other dimensions of d in all other lists

Compute overall score Sc of d

If Sc is among top-k highest scores seen so far

Store d in top-k buffer (break ties arbitrarily)

Stop when k docs are found with overall score $Sc > T$

*See: Optimal aggregation algorithms for middleware

Threshold algorithm (TA): example

- Find top-2 results

dcoID	Tf1
79	0.05
31	0.035
53	0.03
41	0.025
11	0.01

dcoID	Tf2
53	0.06
41	0.04
31	0.028
11	0.02
79	0.01

$T = 0.11$

53: 0.09
79: 0.06

Top-2 result buffer

dcoID	Tf1
79	0.05
31	0.035
53	0.03
41	0.025
11	0.01

dcoID	Tf2
53	0.06
41	0.04
31	0.028
11	0.02
79	0.01

Next threshold smaller than any top-k score → stop!

$T = 0.075$

53: 0.09
41: 0.065
~~31: 0.063~~
~~79: 0.06~~

Top-2 result buffer

No Random Access algorithm (Fagin et al. 2001)

- All inverted lists $L_1, \dots, L_l$ are sorted by tf
- No random access

Precompute and maintain $\min\_1, \dots, \min\_l$, the smallest possible scores from the lists $L_1, \dots, L_l$

Do sorted access in parallel to all lists

Let cdim_i be the last position visited in sorted access in each L_i

Maintain $(\mathrm{cdim}_1.score, \dots, \mathrm{cdim}_l.score)$

For every doc d with some unseen dimension

Compute lower bound Sc^L of Sc by replacing unseen $\mathrm{dim}_i.score$ by $\min\_i$ and upper bound Sc^U of Sc by replacing unseen $\mathrm{dim}_i.score$ by $\mathrm{cdim}_i.score$

Maintain top-k docs with highest Sc^L (break ties using Sc^U scores)

Stop when current Sc^U exceeds smallest top-k score

NRA algorithm: example

- Find top-2 results

dcoID	Tf1	dcoID	Tf2
79	0.04	53	0.06
31	0.035	41	0.04
53	0.03	31	0.028
41	0.03	11	0.02
11	0.01	79	0.01

53: (0.1 – 0.07)
79: (0.1 – 0.05)

Result buffer

dcoID	Tf1	dcoID	Tf2
79	0.04	53	0.06
31	0.035	41	0.04
53	0.03	31	0.028
41	0.03	11	0.02
11	0.01	79	0.01

53: (0.095 – 0.07)
79: (0.08 – 0.05)
41: (0.075 – 0.05)
31: (0.075 – 0.045)

Result buffer

NRA algorithm: example

- Find top-2 results

dcoID	Tf1
79	0.04
31	0.035
53	0.03
41	0.03
11	0.01

dcoID	Tf2
53	0.06
41	0.04
31	0.028
11	0.02
79	0.01

53: (0.09)
79: (0.068 – 0.05)
41: (0.07 – 0.05)
31: (0.063)

Result buffer

dcoID	Tf1
79	0.04
31	0.035
53	0.03
41	0.03
11	0.01

dcoID	Tf2
53	0.06
41	0.04
31	0.028
11	0.02
79	0.01

53: (0.09)
~~79: (0.06 – 0.05)~~
41: (0.07)
~~31: (0.063)~~

Result buffer

Instance optimality of TA and NRA

- Definition
  - For class $\mathcal{A}$ of algorithms and class $\mathcal{D}$ of datasets, algorithm B ∈ $\mathcal{A}$ is instance optimal over $(\mathcal{A}, \mathcal{D})$ if for every A∈$\mathcal{A}$ and every D∈$\mathcal{D}$:

    cost(B,D) ≤c*cost(A,D) + c' ⇔ cost(B,D)= O(cost(A,D))

- It can be shown:
  - For any monotone scoring function, TA and NRA correctly retrieve the top-k results.
  - TA is instance optimal over all algorithms that are based on sorted and random accesses to inverted lists (no „wild guesses").
  - NRA is instance optimal over all algorithms with sequential accesses only.

Implementation issues

- Priority queues
  - Empirically, bounded-size priority queues show better performance than Fibonacci heaps
- Memory management
  - Memory load is very important for efficiency (similarly to scan depth)
  - Early candidate pruning is important
- Hybrid block index
  - Group inverted list entries in blocks and sort blocks by scores
  - Keep entries within a block in docID order
  - After each block read: merge-join first, then update priority queue

"Champion lists" heuristics (Brin & Page 1998)

- All inverted lists $L_1, \dots, L_l$ are sorted by doc authority (e.g., PageRank) scores
- Keep additional lists $F_1, \dots, F_l$ (champion lists) with docs having tf scores above some threshold in each dimension

Compute scores for all docs in $\cap_i F_i$ and keep top-k results;

$Cand := (\cup_i F_i) \setminus (\cap_i F_i)$

For each $d \in Cand$ do

    compute partial score of d

Scan inverted lists L_i in Round-Robin fashion

    if $\dim_i.doc \in Cand$

        add $\dim_i.score$ to partial score of $\dim_i.doc$

    else

        add $\dim_i.doc$ to $Cand$ and set its partial score to $\dim_i.score$

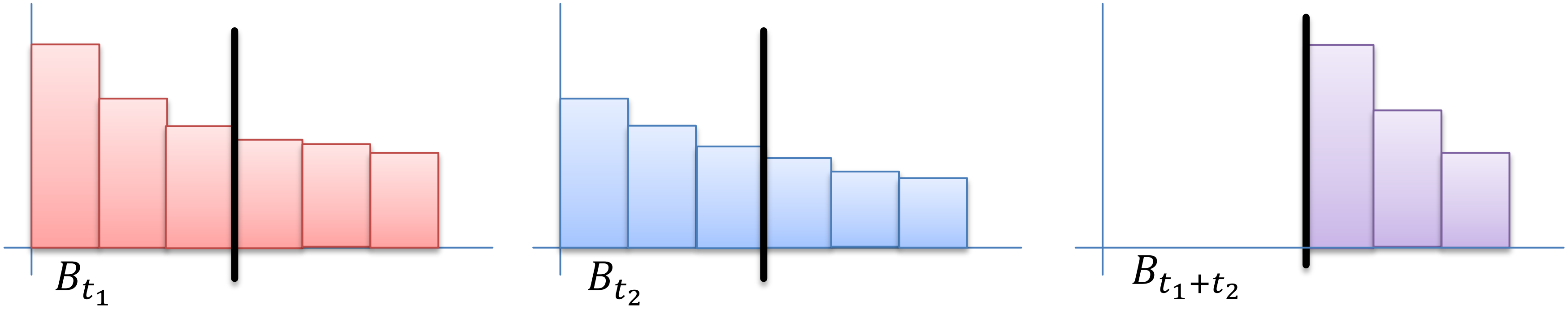
    Terminate when k'> k docs with complete scores are found;

Probabilistic approximate top-k processing

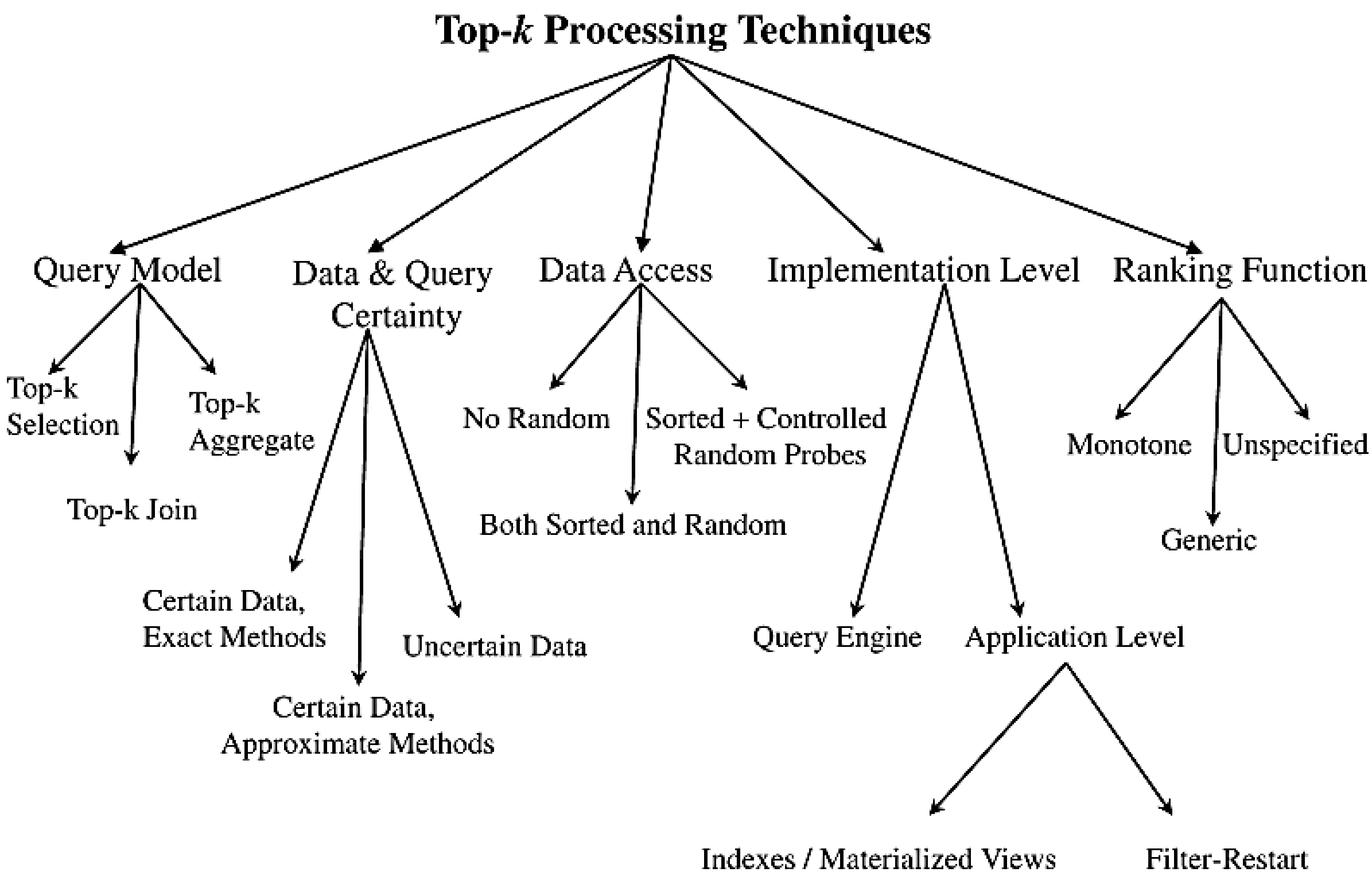
- Makes use of
  - certain score distribution in each of the inverted lists (approximated by histograms)
  - pair-wise convolution of score distributions

$$\sum_{0 \leq i \leq d} B_{t_1}[i].freq * B_{t_2}[d-i].freq = B_{t_1+t_2}[d].freq$$

  - correlation between scores in different dimensions
  - probabilistic inequalities for stopping conditions



Feature overview of top-k algorithms



Source: https://cs.uwaterloo.ca/~ilyas/papers/IlyasTopkSurvey.pdf

Summary

- Distributed index maintenance
  - Horizontal partitioning (by documents)
    - High costs, easy to maintain, scalable, load-balanced
  - Vertical partitioning (by terms)
    - Low costs, maintenance and load-balancing are difficult
- Top-k algorithms
  - Join and sort when list entries are sorted by docIDs
  - When list entries sorted by per-term doc scores:

    Top-k join with score aggregation

    "Champion lists" (uses lists with authority scores)

    Threshold algorithm

    No Random Access algorithm
- Probabilistic approximate top-k processing
  - Estimation of unseen scores by convolution of score distributions in inverted lists