Causal Inference
Theory and Applications in Enterprise Computing

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May 20, 2020

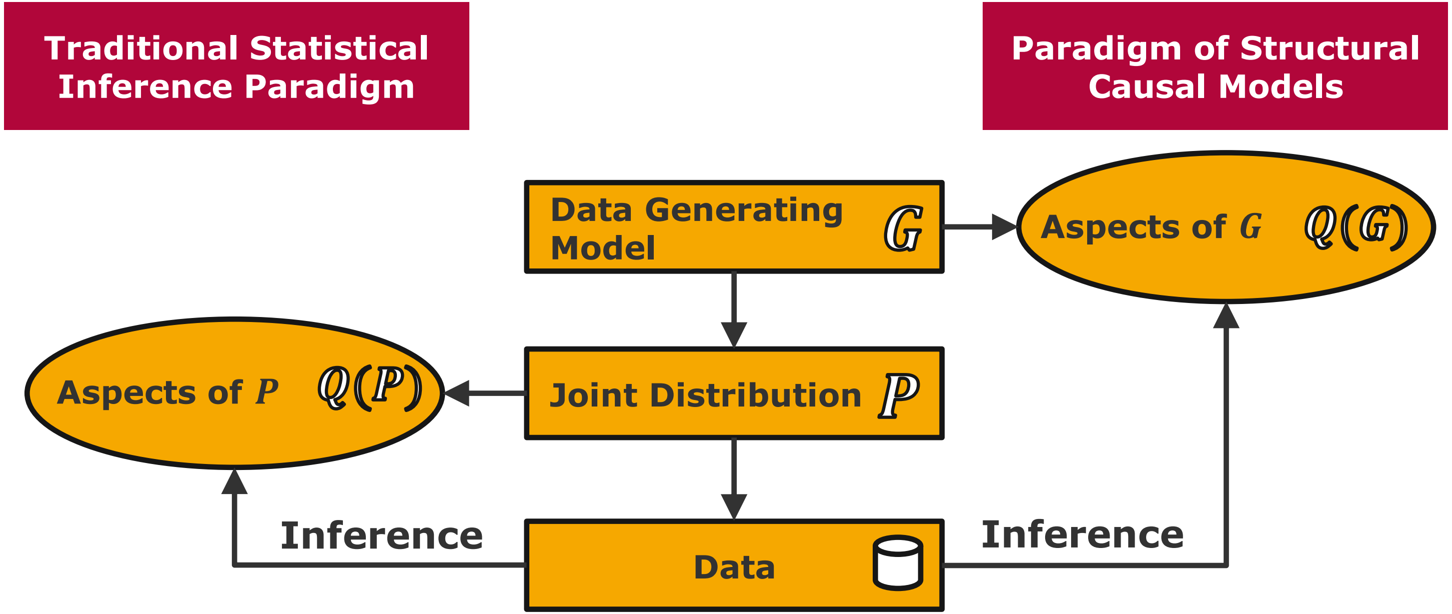
Agenda
May 20, 2020

- **Recap: Causal Inference in a Nutshell**
  - Causal Structure Learning

- **Jupyter Lab**
  1. Causal Inference in a Nutshell - Cooling House Scenario
  2. Causal Structure Learning – Exercises

Recap: Causal Inference in a Nutshell

Recap: Causal Inference in a Nutshell
Concept



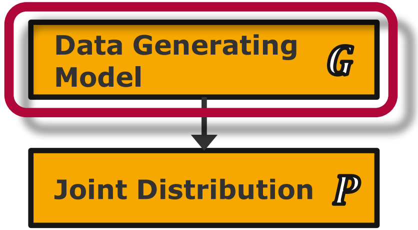
E.g., what is the sailors' probability of recovery when **we see** a treatment with lemons?

$$Q(P) = P(recovery|lemons)$$

E.g., what is the sailors' probability of recovery if **we do** treat them with lemons?

$$Q(G) = P(recovery|do(lemons))$$

Recap: Causal Inference in a Nutshell
Causal Graphical Models



Causal Graphical Model

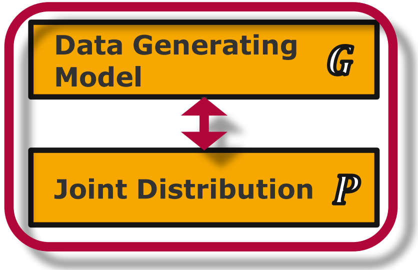
- *Directed Acyclic Graph (DAG)* $G = (V, E)$
  - *Vertices* $V_1, \dots, V_n$
  - *Directed edges* $E = (V_i, V_j)$, i.e., $V_i \rightarrow V_j$
  - *No cycles*
- *Directed Edges* encode direct causes via
  - $V_j = f_j(\mathrm{Pa}(\mathrm{V_j}), \mathrm{N_j})$ with independent noise $N_1, \dots, N_n$

Causal Sufficiency

- All relevant variables are included in the DAG G

Recap: Causal Inference in a Nutshell

Connecting G and P



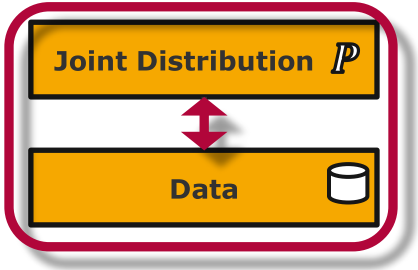
$$(X \perp\!\!\!\perp Y|Z)_G \Rightarrow (X \perp\!\!\!\perp Y|Z)_P$$

- Key Postulate: *(Local) Markov Condition*
- Essential mathematical concept: *d-Separation*
  - Idea: *Blocking* of paths
  - Implication: *Global Markov Condition*

$$(X \perp\!\!\!\perp Y|Z)_G \Leftarrow (X \perp\!\!\!\perp Y|Z)_P$$

- Key Postulate: *Causal Faithfulness*

Recap: Causal Inference in a Nutshell
Connecting P and



Statistical Inference

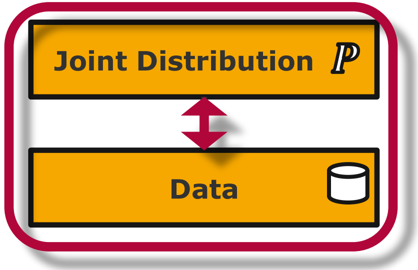
- Essential concept: *Point estimator* $\hat{\Theta}$
  - *Statistic* $g(X_1, \dots, X_n)$ of *random samples* $X_1, \dots, X_n$ to estimate *population parameter* Θ
- Inference: *Statistical Hypothesis Test*
  - *Null Hypothesis* H_0, claim on a population's property initially assumed to be true
  - *Alternative Hypothesis* H_1, a claim that contradicts H_0
  - Rejection criteria for H_0: *c*-value $T(x) > \mathrm{c}$ or equivalently *p*-value $P_{H_0}\big(T(X) > T(x)\big) < \alpha$

$$(\boldsymbol{X} \perp\!\!\!\perp \boldsymbol{Y}|\boldsymbol{Z})_{\boldsymbol{P}} \Leftarrow$$

- Key idea: *Conditional Independence Test*
  - Distribution of $\boldsymbol{V} = \{V_1, \dots, V_N\} \Rightarrow$ dependence measure $T(V_i, V_j, \boldsymbol{S}) \Rightarrow$ hypothesis $H_0: t = 0$

Recap: Causal Inference in a Nutshell

Connecting P and [database icon]



Statistical Inference

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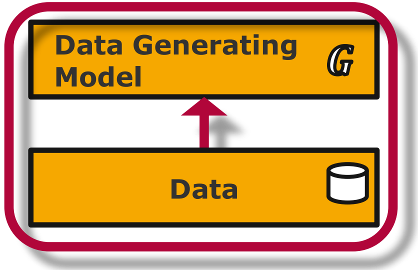
$$(\boldsymbol{X} \perp\!\!\!\perp \boldsymbol{Y} | \boldsymbol{Z})_{\boldsymbol{P}} \Leftarrow \text{[database icon]}$$

- Method: *Conditional Independence Test*
  - Distribution of $\boldsymbol{V} = \{V_1, \dots, V_N\} \Rightarrow$ dependence measure $T(V_i, V_j, \boldsymbol{S}) \Rightarrow$ hypothesis $H_0: t = 0$

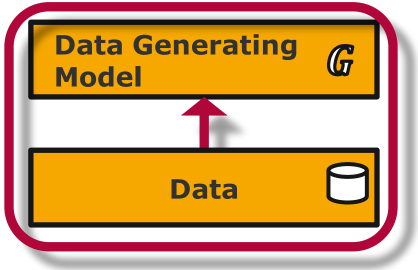
Recap: Causal Inference in a Nutshell
Causal Structure Learning



Causal Structure Learning

- Assumptions: *Causal Sufficiency, Markov Condition, Causal Faithfulness*
- Idea: Accept only those DAG's G for which $(X \perp\!\!\!\perp Y | Z)_G \Leftrightarrow (X \perp\!\!\!\perp Y | Z)_P$
  - Identifies DAG up to *Markov equivalence class* (i.e., same *skeleton* C and *v-structures*)
  - Markov equivalence class uniquely described by *completed partially directed acyclic graph (CPDAG)*
- Basis: V_i and V_j are linked if and only if there is no $S(V_i, V_j)$ s.t. $\left(V_i \perp\!\!\!\perp V_j \middle| S(V_i, V_j)\right)_P$
- Methods:
  - *Constraint-based*: CI testing to derive skeleton together with edge orientation rules
  - *Score-based*: "search-and-score approach"
  - *Hybrid*: Constraint-based skeleton derivation and score-based edge orientation

Recap: Causal Inference in a Nutshell
Causal Structure Learning



PC Algorithm

- Concept:
  - *Skeleton discovery*: Iterative CI testing given increasing adjacent $S(V_i, V_j)$
  - *Edge orientation*: Deterministic orientation rules implied by Markov equivalence class
- Properties:
  - Polynomial complexity given sparse graphs G (exponential in worst case)
  - Asymptotic consistency (under technical assumptions) $\Pr(\hat{G} = G) \to 1 \quad (n \to \infty)$
  - Extensions allow for weaker faithfulness, latent variables, cycles, etc.

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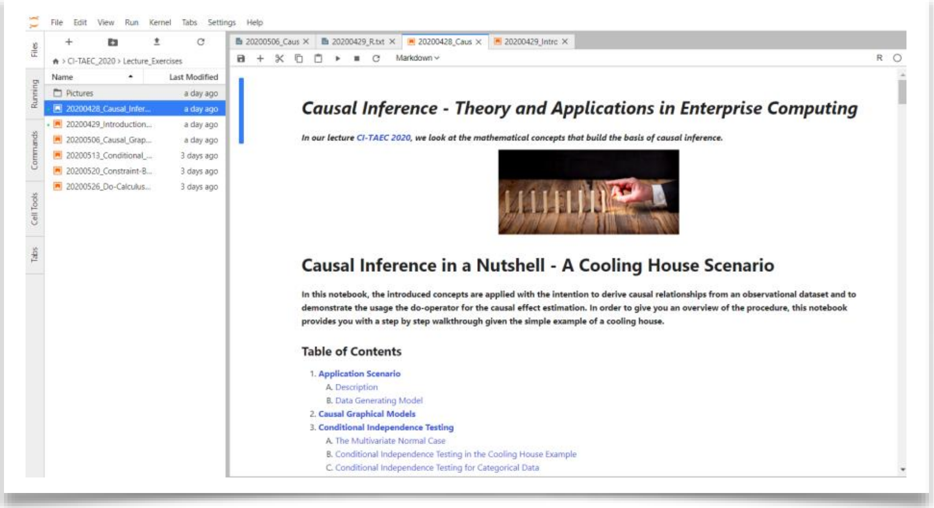
Jupyter Lab

Jupyter Lab
Causal Inference in a Nutshell - Cooling House



Topics

- CSL in the Cooling House Example

Jupyter Lab
CI Tests - Exercises

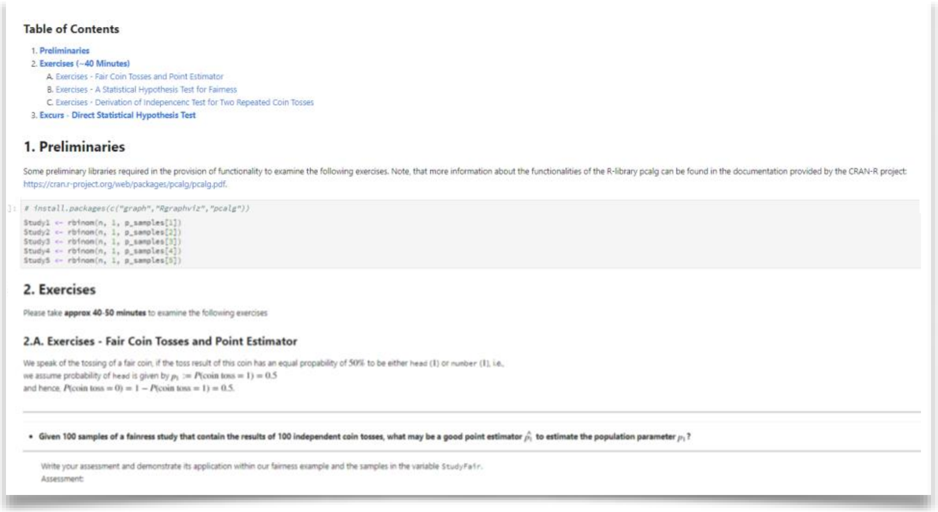


Table of Contents

1. Preliminaries

Some preliminary libraries required in the provision of functionality to examine the following exercises. Note, that more information about the functionalities of the R-library pcalg can be found in the documentation provided by the CRAN-R project: https://cran.r-project.org/web/packages/pcalg/pcalg.pdf.

```
# install.packages(c("graph","Rgraphviz","pcalg"))
Study1 <- rbinom(n, 1, p_samples[1])
Study2 <- rbinom(n, 1, p_samples[2])
Study3 <- rbinom(n, 1, p_samples[3])
Study4 <- rbinom(n, 1, p_samples[4])
Study5 <- rbinom(n, 1, p_samples[5])
```

2. Exercises

Please take **approx 40-50 minutes** to examine the following exercises

2.A. Exercises - Fair Coin Tosses and Point Estimator

We speak of the tossing of a fair coin, if the toss result of this coin has an equal propability of 50% to be either head (1) or number (1), i.e.,
we assume probability of head is given by $p_1 := P(\text{coin toss} = 1) = 0.5$
and hence, $P(\text{coin toss} = 0) = 1 - P(\text{coin toss} = 1) = 0.5$.

- **Given 100 samples of a fairness study that contain the results of 100 independent coin tosses, what may be a good point estimator $\hat{p}_1$ to estimate the population parameter p_1?**

Write your assessment and demonstrate its application within our fairness example and the samples in the variable StudyFair.
Assessment:

Exercises

- Markov Equivalence Class
- Causal Sufficiency
- Causal Faithfulness

Thank you
for your attention!