Homework DDDM SS2020 (until July 31)

Exercise 1 (Linear Programming)

Take a Sudoku riddle and find its solution using Linear Programming.

(a) Formulate a general model, where the constraints guarantee a feasible solution.

(b) Solve a specific example and output the solution in an appropriate way.

(c) Verify the uniqueness of your example's solution (using LP).

Exercise 2 (Dynamic Programming)

We want to sell event tickets using Dynamic Programming. We seek to find a pricing policy that optimizes expected profits. We have *N*=40 tickets and *T*=100 periods of time. Tickets cannot be sold after *T*. We do not use a discount factor and there is no salvage value for unsold items. We consider the following demand probabilities, i.e., $P_t(1,a) := \max(0, 1-a/50)$ and $P_t(0,a) := 1 - P_t(1,a)$, $a \in A := \{0,1,2,...,50\}$, $t = 0,1,...,T$.

(a) Formulate a general model to sell tickets under given *N*, *T*, and demand probabilities P_t.

(b) Solve the given example and output the solution in an appropriate way.

(c) Simulate 1000 runs of applying the optimal policy over *T* periods. Show the distribution of realized total profits of these 1000 runs. Compare the mean with the value function.

Exercise 3 (Regressions via Nonlinear Programming)

We want to extend the least squares example (OLS) discussed in the course, see *linear regression*.

(a) Normalize the values of the *K* explanatory variables $x_{i,k}$, $i = 1,...,N$, $k = 1,...,K$, with *N* observations to $\tilde{x}_{i,k} := \left(x_{i,k} - \overline{x}_k\right)/\sqrt{\sigma_k}$ using the mean $\overline{x}_k := 1/N \cdot \sum_{i=1,...,N} x_{i,k}$ and the variance $\sigma_k := 1/N \cdot \sum_{i=1,...,N} (x_{i,k} - \overline{x}_k)^2$ for all *K* dimensions.

(b) Perform a Lasso Regression (L1 regularization) using the extended objective, e.g., $\alpha := 1$,

$$\min_{\beta_k \in \mathbb{R}, k=1,...,K} \left\{ \frac{1}{N} \cdot \sum_{i=1}^{N} \left(\vec{\tilde{x}}_i{}' \vec{\beta} - y_i \right)^2 + \alpha \cdot \sum_{k=1}^{K} |\beta_k| \right\}.$$

(c) Perform a Ridge Regression (L2 regularization) using the extended objective, e.g., $\alpha := 1$,

$$\min_{\beta_k \in \mathbb{R}, k=1,...,K} \left\{ \frac{1}{N} \cdot \sum_{i=1}^{N} \left(\vec{\tilde{x}}_i{}' \vec{\beta} - y_i \right)^2 + \alpha \cdot \sum_{k=1}^{K} \beta_k{}^2 \right\}.$$

To hand in: executable files and a short documentation of the results.